THE

Mathematics Student

A Quarterly Dedicated to the Service of Students and Teachers of Mathematics in India.

VOL. X—1942

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Printed by REV. FR. A. SUSAI REGIS,
Supt., St. Joseph's Industrial School Press, Trichinopoly

AND

Published by S. MAHADEVAN, M.A.,
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1944

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Page	*Line*	*For*	*Read*
40	1B†	Zassenkaus	Zassenhaus
41	23	p_α th	p^α th
,,	9B	ω^μ	ω_μ
,,	9B	p^{ω^α}	p^{ω_α}
,,	5B	$\mu\omega_1$	μ_{ω_1}
,,	3B	$\mu\gamma$	μ_ω
70	13B	${}_{n-1}C_1$	${}_{n-i}C_1$
,,	10B	l	1
71	22	=	−
,,	24	K	k
74	12	$\mu's$ can also	μ'_s can also
98	13B	A Supplementary to	A Supplement to
104	(Question 1811)	the 9 points A_{23}, B_{23}, C_{23}, D_{23}, A_{13}, B_{13}, C_{13}, A_{12}, B_{12}	the 12 points A_{ij}, B_{ij}, C_{ij}, D_{ij} $(i, j=1, 2, 3, 4)$
155	3	$M_{\nu-1_{r+1}}$	$M_{\nu_{r+1}-1}$
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157	4B	and $n'' \nu$	and n_ν''
159	5B	m_r'	m'_{ν_r}
160	20	${}_{\sigma\nu}a'_\mu$	$\sigma_\nu a'_\mu$
161	2B	$a_r' \leqslant a_r''$	$a_r' > a_r''$
,,	1B	$a_\nu' \leqslant a_\nu''$	$a_\nu' \geqslant a_\nu''$
176	13	**Notes on Ellipse**	**A Note on the ellipse**

† To be read "line 1 from the bottom of the page."

INDIAN MATHEMATICAL SOCIETY

The Narasinga Rao Medal For Mathematical Research

This is a gold medal which will be awarded periodically by the Indian Mathematical Society out of the interest on an endowment fund created for the purpose by Dr. A. Narasinga Rao of the Annamalai University. The object of the award is to encourage mathematical research and to direct attention to modern topics which have not received sufficient attention in this country.

The medal will be awarded to the best solution or contribution towards the solution of a specific problem in mathematics. The problem will be announced in the press and in the periodicals published by the Society, and a period of at least 18 months will be given before solutions are called for. A special Committee of the Indian Mathematical Society is in charge both of the selection of the problem and the adjudication of the award, the decision of the committee being final.

To be eligible for competing for the medal, the competitor must be of Indian domicile, and a member of the Indian Mathematical Society, both at the time of submitting his thesis and at the time of award.

The first problem has been selected and is given below :—

Problem

"It is required to characterise and study the strongest topology that can be imposed on the linear space of those real-number sequences $a = (a_1, a_2, a_3, \ldots)$, *that have a convergent sum* $S(a) = a_1 + a_2 + \ldots$ *to infinity, so that* $S(a)$ *is a continuous functional of* a."

Explanation

The sought topology T must be a group-topology, so that for any two sequences a, b of the space. the sequence $a-b$ must be a continuous function of the two variables a,b. It is also clear that T must be "weaker" than the direct product topology of a sequence of real-number spaces, so that any element a_n of a will be a continous functional of a, in the topology T. It is suspected that T is not metrisable, but is a uniform-structure topology in the sense of Andre Weil.

This is, in a sense, a key-problem in the subject of linear and function-spaces. The space of real-number sequences with convergent sum, is not one of the regular kinds of spaces listed by Banach in his *Linear operators,* but is typical of the more troublesome intermediate kinds of space which arise. The solution of this problem will serve to indicate the nature of these intermediate spaces generally.

If we confine ourselves to the Lebesgue Integral, there is no exact integral analogue to the above problem in function-spaces; for, the Lebesgue integral is always absolutety convergent. We can, however, formulate an analogous problem for function-spaces with a non-absolutely convergent integral like the Denjoy integral; *viz.*

"If F is the space of all Denjoy-integrable functions defined in a finite interval (0,1), to find the strongest topology on F, for which the Denjoy-integral of any function is a continuous functional of that function."

Remarks on this strongest topology T may be made, analogous to these already made in the case of sequences.

As regards references likely to be useful in connection with the Prize Problem, the idea of strong and weak topologies will be found referred to in Alexandroff and Hopf: *Topologie* and Banach's *Theorie des operations lineaires.*

The last date for the submission of the solutions of the Prize Problem will be the 31st July, 1945. Theses should be sent by registered post to Dr. A. Narasinga Rao, Annamalai University, Annamalainagar.

13 SEP 1942

TWELFTH CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY,

HELD AT ALIGARH, ON THE 27th, 28th & 30th DECEMBER 1941.

Sitting :— C. P. S. Menon (*Dehra Dun*), S. M. Shah (*Aligarh*), C. N. Srinivasiengar (*Bangalore*), A. N. Singh (*Lucknow*), F. W. Levi (*Calcutta*), Sir Ziauddin Ahmad (*Aligarh*), R. Vaidyanathaswamy (*Madras*), A. B. A. Haleem (*Aligarh*), Khan Bahadur A. M. Kureishy (*Aligarh*), S. W. Shiveshwarkar (*Lucknow*), A. Narasinga Rao (*Annamalainagar*), Ram Behari (*Delhi*), D. D. Kapadia (*Poona*), M. M. Sharif (*Aligarh*), S. M. Kerawala (*Aligarh*), Mohammad Ozair (*Aligarh*)

Standing 1st Row :—Sahib Ram (*Lahore*), Hansraj Gupta (*Hoshiarpur*), S. R. Das (*Delhi*), B. R. Pasricha (*Lucknow*), Omar Ali Siddiqi (*Aligarh*), V. Rangachariar (*Patna*), P. D. Shukla (*Lucknow*), R. S. Varma (*Lucknow*), R. D. Misra (*Lucknow*), B. Ramamurti (*Ajmer*), R. Krishnamurti (*Hyderabad Dn.*), Ekanath Banerji (*Cawnpore*), S. C. Sircar (*Bombay*), D. N. Moghe (*Bombay*), G. Sarwar (*Aligarh*), M. Ziauddin (*Lahore*), D. J. Madan (*Karachi*), S. K. Abhyankar (*Gwalior*), Abdullah Butt (*Aligarh*), Abdul Jabbar (*Calcutta*), M. Zakiuddin (*Aligarh*)

Standing 2nd Row :—S. S. Pillai (*Travancore*), Ram Ballabh (*Lucknow*), Ahmad Abbas (*Hyderabad Dn.*), Prithvi Nath Sharma (*Delhi*), A. Vindhyachal Prasad (*Chapra*), S. Nageswaran (*Delhi*), D. R. Kaprekar (*Devlali*), G. R. Seth (*Delhi*), F. C. Auluck (*Lucknow*), G. S. Diwan (*Bombay*), S. M. Shafi (*Aligarh*), Tahir Rizir (*Aligarh*), Zakiuddin (*Aligarh*), Hari Shanker (*Delhi*), D. D. Kosambi (*Poona*), K. R. Gunjikar (*Bombay*), Banwari Lal (*Meerut*), P. B. Bhattacharya (*Delhi*)

Standing 3rd Row :—Nazir Ahmad (*Delhi*), Hashim (*Peon*), Abdul Hafeez (*Aligarh*), P. C. Mital (*Lucknow*), P. N. Saxena (*Lucknow*), Afzal Ahmad (*Hyderabad Dn.*), Misbahauddin Khan (*Aligarh*), M. L. Widhani (*Aligarh*), Mohsin Siddiqi (*Aligarh*), Mohd. Shabbar (*Aligarh*), Mufizul Islam (*Aligarh*), K. Ramachandran (*Hyderabad Dn.*)

THE MATHEMATICS STUDENT

Volume X] MARCH 1942 [Number 1

THE TWELFTH CONFERENCE

OF THE

INDIAN MATHEMATICAL SOCIETY

The Indian Mathematical Society held its Twelfth Conference at Aligarh at the invitation of the Muslim University on the 27th, 28th and 30th of December 1941.

The Conference opened at 11 A.M. on Saturday the 27th December in the Strachey Hall. The delegates were welcomed by Prof. A. B. A. Haleem, Pro-Vice-Chancellor, and the Conference was inaugurated by Sir Ziauddin Ahmad, Vice-Chancellor of the Muslim University, Aligarh. Dr. Ram Behari (Delhi), the Secretary of the Society then read the Report of the Society's activities, and this was followed by the Presidential Address by Dr. R. Vaidyanathaswami (Madras) President of the Society.

Besides the reading of the papers, on the 28th and 30th December, there were three symposia—one on "Fourier integrals and transforms" with Dr. R. S. Varma (Lucknow) presiding, a second on "Group Theory" under the Chairmanship of Dr. F. W. Levi (Calcutta) and a third on "The Origin of the Solar System" initiated and presided over by Prof. A. C. Banerji (Allahabad).

There were 2 evening lectures, one by Dr. A. Narasinga Rao (Annamalainagar) on "Mathematics and Modern Warfare" and the other by Dr. B. Ramamurti (Ajmer) on "How to make the teaching of Mathematics interesting" Prof. D. D. Kosambi (Poona) addressed the Students of the Mathematics Association of the Muslim University on the evening of the 27th December.

As usual there was a Business Meeting of the Society and a *Discussion on* the Teaching of Mathematics in Schools and Colleges, initiated by Sir Ziauddin Ahmad.

While the serious side of the Conference thus received due attention, the social side was by no means neglected. The guests were comfortably lodged and fed more sumptuously than was good for their health. Specially worthy of mention were the splendid Tea on the 27th and Lunch on the 30th to which the guests were invited by Sir Ziauddin Ahmad and the rich Dinner on the 28th provided by Seth Phoolchandji of Aligarh City.

Special features which made the Aligarh conference noteworthy were the Annual Session of the Benares Mathematical Society held for the first time

in collaboration with the Indian Mathematical Conference, and the announcement of a prize to be awarded once in two years for the encouragement of Mathematical research which Prof. A. Narasinga Rao offered to institute at his own expense. The details of the prize are under consideration by the Committee of the Indian Mathematical Society.

Reception Committee

Chairman

Lt.-Col. Dr. Sir Ziauddin Ahmad, M.A., Ph.D., D.Sc., Kt., C.I.E., M.L.A. (Central), Vice-Chancellor, Muslim University, Aligarh.

Vice-Chairmen

Prof. A. B. A. Haleem, B.A. (Oxon.) Bar-at-Law, Pro-Vice-Chancellor, Muslim University, Aligarh.

Haji Mohammad Swaleh Khan Sahib Sherwani.

Khan Bahadur A. M. Kureishy, M.A., Chairman, Department of Mathematics, Muslim University, Aligarh.

Treasurer

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Secretary

S. M. Kerawala, Esqr., M.A. (Cantab.)

Joint-Secretaries

Abdullah Butt, Esqr., M.A.

Omar Ali Siddiqi, Esqr., M.A.

Welcome Address

BY

PROF. A. B. A. HALEEM, *Pro-Vice-Chancellor*

(27th December 1941)

Mr. President and Delegates to the Indian Mathematical Conference:

I deem it a very pleasant duty and a great privilege to welcome on behalf of the Aligarh Muslim University the members of the Indian Mathematical Conference. The invitation to the Conference was extended in fulfilment of the wish expressed by the late Sir Shah Mohammad Sulaiman, the then Vice-Chancellor of the University, and it is a matter of grief to us all that that distinguished scientist is no longer in our midst. The place vacated by the death of Sir

Shah at the early age of 54 was filled up by Sir Ziauddin Ahmad, who has won distinction and fame in various fields of activity, including the field of work to which the members of this Conference have dedicated themselves. It is but seldom in the life even of an educational institution like ours that such a gathering of the leading Mathematicians of India takes place, and we are grateful to the President and the Secretary of the Conference for having kindly accepted our invitation thus enabling us to establish a contact between the delegates on the one hand and the staff and students of this University on the other.

The University to which I am welcoming you today was founded with the object of combining the study of Western Arts and Sciences with Eastern learning. Though called into being for the purpose of meeting the requirements of one of the two major peoples of this country, it has always managed to attract students belonging to all creeds and faiths in large numbers. Drawing its alumni from all the provinces of India and even from foregin lands, it has aimed, not without a large measure of success, at developing a broad outlook on life and a spirit of large-hearted toleration. The study of the science of Mathematics has always been duly stressed at this University and our Mathematics Department has been staffed by a succession of able men. The first Professor of Mathematics in the Mohammadan Anglo Oriental College, which was transformed in the year 1920 into the Aligarh Muslim University, was Babu Jadav Chandra Chakaravarti, a person well known in the mathematical world of India, who is still remembered with affection by his pupils. On his retirement Babu Chakaravarti was succeeded by Doctor (now Sir) Ziauddin Ahmad, who has ever since maintained a close connection with our Department of Mathematics and in spite of his multifarious activities, still lectures to the students of the Department in the capacity of Honorary Professor. Amongst those who have served the Department at various stages in the past, the names of Dr. O. H. Malik, Dr. Vijyaraghavan, Dr. Andre Weil and Mr. Kosambi may especially be mentioned.

I should like to avail myself of this opportunity, to draw your attention to the need of evaluating the contributions made to the advancement of Mathematics by the Hindu and Muslim Mathematicians.

Important contributions to the progress of the science—in the domain of pure as well as applied mathematics—were made by the Islamic world, particularly under the early Abbaside Caliphs, and the achievements of the Muslim Mathematicians present a vast and attractive field of work for the researcher. The exploration of this field, however, is no easy task and requires close and prolonged co-ordination between a body of Mathematicians and Arabicists.

The method of teaching the subject—particularly in our schools—is also one which deserves some attention from a Conference, comprising in its ranks Mathematicians and educationists. It is a regrettable fact that our system of instruction fails to inspire in a great majority of the pupils that love for the science of Mathematics which is the only way of promoting research at a higher

stage. The subject is taught from practically the lowest classes right up to the highest and a great deal of time and attention is devoted to it, but the result is not in keeping with the effort put forth. The question is one which needs examination and we shall have to revise and improve our system of instruction before we can hope to obtain results in consonance with our efforts and impart mental discipline which is the main justification for retaining the subject in the school curriculum.

In conclusion, I extend to you once more a most cordial welcome and hope that under the able guidance of an eminent Mathematician like Dr. Vaidyanathswami your deliberations will yield results of great value to the country and to the world at large.

Opening Speech

BY

LT.-COL. DR. SIR ZIAUDDIN AHMAD, *Vice-Chancellor, Muslim University*

(27th December 1941)

MR. PRESIDENT AND THE MEMBERS OF THE MATHEMATICAL CONFERENCE,

I first thank the members of the Mathematical Conference for their kindness in accepting the invitation of this University for holding their 12th session at Aligarh and I welcome the delegates who have made it convenient to respond to the invitation.

Our invitation to the Mathematical Conference was initiated by the late Sir Shah Muhammad Sulaiman whose loss on the present occasion we all deplore. In him the country lost a great jurist, a devoted scientist and an expert educationist. His researches and the incomplete work which he left may come up for discussion before this Conference. I am sure that I express the feelings of all the members when I say that we all miss at this Conference his personality at whose initiation we have assembled here today.

The researches in Mathematics are comparatively more difficult than those in other branches of science. In physical sciences, negative results have some value, but in Mathematics the negative results have no place. In every science the application of general principles to particular cases and their verification by using different sets of substances has some research value.

The chief difficulty in the Mathematical research is the absence of a well-equipped library supplied with current and back numbers of important periodicals. Some libraries have made individual attempts to collect books dealing with certain aspects of Mathematics, but no library can be said to be complete. It was suggested sometime ago that the Government of India should build up a well-equipped library either at Delhi or in Calcutta and that it should lend out books and periodicals to the University and College libraries. Every student of Mathematics who comes back after study in a European

University has certain problems on which he would like to continue his research, but due to the absence of opportunities he is unable to continue his research.

On account of the absence of contact between the teachers of different Universities and the want of up-to-date literature, research workers are very much handicapped in this country. This difficulty is more noticeable in problems affecting more than one science like the constitution of an atom. It is primarily a chemical problem but most of the researches are based on physical experiments and which are subsequently deduced by mathematical reasoning.

One would like to expect that the laws of nature governing the movements of bodies whether in a solar system or elsewhere must be the same. The use of spectroscopic measurement in recent years has attained a degree of accuracy which older methods did not possess. The minute measurement has revealed that Newton's law of gravitation which was universally adopted on account of its simplicity has failed to explain the new phenomena. Many physicists and Mathematicians have long been contemplating that Newton's law of gravitation was the first approximation of some other law which perhaps could not be expressed in terms of the functions already known to us. Einstein was the first who modified the law by introducing the fourth dimension. Whatever the merits of his discoveries may be, he has left a permanent contribution not only in Mathematics but in Philosophy that space is not absolute. The researches of Michelson, Morley, Fitzgerald and others have proved that in all measurements, the motion of the observer should also be considered and they were led to the conclusion that the notions of time and space are inter-dependent. Einstein succeeded in explaining (1) the advance of the perihelion of mercury which is 43′ per century, (2) spectral shift of light, and (3) deflection of light which Newtonion law had failed to explain.

The late Sir Shah Sulaiman started by adding the inverse of the fourth power to Newton's law of gravitation, and he took the co-efficient in the form in which it was given by Einstein. By his method he obtained a correction for the deflection of light which is fifty per cent more than the quantity arrived at by Einstein and even his figure is slightly short of the observation. Just before his death he was studying his own law, but could not complete his work.

Newton's law singularly fails in the motion of electrons, and especially when we deal with a velocity comparable to that of light. These are now explained by introducing the notion of quanta and by simplifying the differential equations by the use of highly complicated symbols. This has placed us in the same position in which celestial mechanics drifted when they attempted to explain heavenly phenomena by epicycles and hypocycles. The theory necessitated the addition of one more circle in order to explain a new phenomena discovered by more minute observations, so much so that King Alfonso once said that he would have created the universe in a simpler form. The universe is simple but our explanations are complicated. Lord Kelvin

once said that the discoveries in the physical sciences are forcing us to believe that we are on the verge of a great discovery which will explain the existing phenomena in a simpler manner.

I am glad that this Conference has invited a discussion on the teaching of Mathematics in Universities and Colleges. Our mathematical study is suffering for two main reasons:

Firstly, we have forced into the curriculum a number of subjects discovered in recent years and whose knowledge is essential at the present day without sufficient scrutiny and without modifying suitably the school syllabus. Thus the study of Mathematics in higher classes has a tendency to become more superficial, and this is enhanced by teaching Mathematics through languages which are not fully developed for scientific instruction. Secondly, we have not provided suitable courses of Mathematics for the students of Physics, and this is prejudicing the coordination of Physics and Mathematics.

Mr. President, the late Sir Shah Sulaiman planned the translation of the great work of El-Beruni, called *Kanoon Masoodi* written in 1038 whose publication had been continuously pressed ever since its publication was demanded by the French Academy of Science. El-Beruni knew Greek and Sanskrit and he studied Greek and Indian astronomy from original sources. Professor Sachau who was a great authority on El-Beruni and who published two of his important works, History of India and Chronology of Oriental people, once said that El-Beruni was the greatest intellect that ever lived on this globe. His book *Kanoon Masoodi* can be edited only by a ripe scholar of Arabic language thoroughly familiar with both ancient and modern astronomy, a rare combination.

I published a paper in 1905 on the third book of *Kanoon Masoodi* dealing with Trigonometry and drew attention to the following facts: regarding the Author:

(1) He is the first Mathematician who wrote an independent book on Trigonometry.

(2) He is the only Arab astronomer who took unity instead of 60 for the radius of the circle, and he defined six trigonometrical ratios by the lengths of straight lines. By this definition the geometrical properties of triangles can be interpreted in trigonometrical identities.

(3) He calculated the sine tables for every increase of 1° correct to seven places of decimals and tangent tables correct to five places of decimals.

(4) He calculated the value of sine 1°, or π correct to fourteen places of decimals.

(5) Since the trisection of an angle is necessary for the calculation of sine 1° he gave 12 mechanical methods for trisecting by trial a given angle, which he naturally knew was impossible by theoretical geometry of lines and circles.

(6) He proved the six formulae of spherical right angled-triangles and also gave for the first time the proof of the formulae $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

(7) He gave methods for finding the values of sine and tangent for intermediary angles. He employed the principle of proportionate parts for rough calculation, but for more accurate calculations he used second differences and he gave geometrical proof of the interpolation formula known after the name of Newton.

(8) He first introduced the notion of 'function' and generalised his interpolation formula to any function calculated for regular increase or decrease in the values of the variable.

(9) He attempted to calculate the radius of the earth by finding the longitude of the place at which the peak of the mountain *Hindukush* was just visible at the horizon. He gave in the same book his own observations and calculated the value of the radius. El-Beruni was not familiar with atmospheric refraction and his results were not accurate. He realised the weakness of his results and throughout his book he accepted the length of 1° of the earth obtained by *Habash* and *Elfraghan* by actual measurements.

The third book of *Kanoon Masoodi* has now been published and we know the contributions of El-Beruni to Trigonometry. One chapter of his book dealing with Geography has just been published by the Archaeological Department of the Government of India. Sir Charles Elliot wrote a memorandum on this subject which is now in the British Museum. El-Beruni's researches in Celestial Mechanics and Physical astronomy are not fully known. We have glimpses of his contributions to other branches of astronomy, a few of which may be described here.

(1) He was familiar with Telescope which is supposed to have been invented by Galileo.

(2) He has given the right ascension and declination of 1029 stars by his own observations.

(3) In Chapter II Book IV he described the nebulae in detail and he described them as cluster of stars.

(4) In Book IV he mentioned that several stars have shifted from the position given by ancient astronomers and mentioned particularly the star in the heart of Leo which he said was not in its old position.

El-Beruni begins the discussion on every new topic by giving an account of the researches of his predecessors, Greeks, Arabs and Indians on the subject, before expounding his own views. Hence the correct history of astronomy cannot be apprehended until *Kanoon Masoodi* of El-Beruni is published in modern notation.

A notable example is the discovery of the secular acceleration of the moon's mean motion which Halley discovered by comparing the tables of Ibne Yunus and Ptolemy and which he fixed at 10″ per century. It was left to Adam and De Launy to examine the cause of this secular variation and to calculate more accurately its value, which according to them was 6″·176 per century. Newcomb observed that Arabian observations are far more reliable than those of Ptolemy and found the variation to be 7″ by comparing modern observations with the observations of Ibne Yunus. The publication of *Kanoon Masoodi* will throw a great light on the influence of Indian and Greek astronomy on the researches of the Arabs.

Mr. President, I request you to commence the formal proceedings of the Conference.

SECRETARY'S REPORT

BY

DR. RAM BEHARI, *Secretary of the Society*

1. On behalf of the Managing Committee of the Indian Mathematical Society, it is my pleasant duty to extend to you all, a hearty welcome to this Conference.

We are specially grateful to you, Sir, for the trouble you have taken, in spite of your multifarious duties to come to inaugurate our deliberations.

2. It is with deep sorrow that we have to record the heavy loss which the Society has sustained in the deaths of :—

(i) Prof. M. T. Naraniengar, formerly Professor of Mathematics at the Central College, Bangalore, who died at Bangalore on the 9th October, 1940. Professor Naraniengar was the Foundation Editor of the Journal of the Indian Mathematical Society and laboured unceasingly and almost single-handed for twenty years to make it a worthy embodiment of the creative activity of India in the field of higher Mathematics. He was President of the Society during the years 1930-32 and was universally loved and respected for his high sense of duty and the simplicity and saintliness of his character.

(ii) Sir Shah Mohammad Sulaiman, formerly Vice-Chancellor of the Aligarh University and Judge of the Federal Court, who died at Delhi on the 12th March 1941. Sir Shah Sulaiman was an active member of the Managing Committee of our Society, and about a fortnight before his death, as Vice-Chancellor of this University, he had invited our Society to hold its Twelfth Conference here. He was an exceptionally gifted lawyer, a keen Mathematician and a great patron of learning, and was admired and loved by all fellow-workers in the field of Science or Law.

3. Our Society was founded by the late V. Ramaswamy Aiyar and a group of enthusiastic associates in 1907 to promote the advancement of

Mathematical Study and research in India, and since then it has been growing steadily in strength and usefulness under the wise guidance of its successive presidents.

During the time that has elapsed since we last met at Hyderabad-Deccan in December 1939, the Society has fairly maintained its strength. 19 new members have been enlisted including one life member.

4. Our Society has three main activities viz., organizing the biennial Conferences, the publications of two Journals and the maintenance of a library of advanced books and journals which are circulated to members who ask for them.

The first conference of the Society was held at Madras in December 1916. Since then we have had conferences at Bombay, Lahore, Poona, Bangalore, Nagpur, Trivandrum, Delhi, Lucknow and Hyderabad (Deccan). We are now holding our Twelfth Conference in this centre of learning under the auspices of the Muslim University.

It is very encouraging to note that in spite of anxious times through which our country is passing on account of war which has spread to our very doors, our members and well-wishers have assembled here in large numbers to make this conference a success by their presence and mathematical contributions. More than 50 papers have been communicated to this conference. We have also been able to arrange three symposia on modern research topics, three popular lectures on interesting subjects, and a discussion on the courses and methods of teaching mathematics.

At the business meeting of the last conference at Hyderabad-Deccan, a committee consisting of Dr. R. Vaidyanathaswami, Dr. A. N. Singh and Professor N. R. Sen was appointed to consider means for securing a closer co-operation between the Benares, Calcutta, and the Indian Mathematical Societies. It is gratifying to note that the Benares Mathematical Society will be holding its annual meeting during our conference session tomorrow.

At the suggestion of the Hyderabad conference, a committee was appointed to examine the place of Mathematics in Higher Competitive Examinations. A memorandum was sent to the Secretary of the Federal Public Service Commission and a very satisfactory reply was received from him.

5. The Society continues to publish two quarterlies. *The Journal of the Indian Mathematical Society* is now managed by a Board of Editors with Dr. Vaidyanathaswami as Chairman and Dr. C. N. Srinivasiengar as the Secretary. It publishes only original papers and caters for the needs of workers in the field of advanced mathematics. The other Journal, under the title, '*The Mathematics Student*' is edited by Dr. A. Narasinga Rao of the Annamalai University and is dedicated to the service of students and teachers of Mathematics in India. It serves as a link between higher mathematics

and the sphere of collegiate mathematics, and also publishes 'News items with the object of keeping the readers in touch with one another and with the leading events in the mathematical world in India and abroad.

During the period 1940-41, 65 papers were received for publication in the *Journal*. It is gratifying that the quality and quantity of mathematical research work submitted for publication is fast improving. Owing to the large number of papers awaiting publication, the size of the Journal was increased from about 36 pages to 48 pages per issue, with effect from December 1940; the get-up and type were also improved, but the new bolder types employed have more or less neutralised the increase in the number of pages and the question of increasing the size of the Journal requires consideration again.

The large number of papers awaiting publication is partly due to the War, since a number of papers which would have gone out to foreign periodicals are now being published in our Journal, but there is every reason to hope that the flow of good papers to our Journal will be maintained at the present level, even after the War.

The *Mathematics Student* continues to serve the student world by critical treatment of collegiate mathematics and by drawing attention to interesting topics and research problems. Undergraduate students and research scholars have been contributing regularly to its columns though the response has not been as plentiful as was hoped. Since it serves as a training ground for young mathematicians in acquiring the art of presentation of their investigations clearly and without the central idea becoming obscured by details, many of the papers have had to be revised (and sometimes rerevised) before publication. The *Student* has appeared without a break four times a year, though somewhat after its due date.

Consequent on the war, the cost of printing is continuously going up. On the other hand, in spite of persistent efforts, it has not been possible to increase our income proportionately. Authors of papers are therefore requested to make them as brief as possible; they are also requested to return the proof-sheets of their papers promptly to avoid unnecessary delay in their publication.

The Society is grateful to all the referees, who have most willingly and at great personal sacrifice assisted the Society with their expert advice on papers sent to them for opinion.

6. The Library of the Society continues to be under the charge of Professor R. P. Shintre and Professor D. D. Kosambi, of the Fergusson College, Poona. Owing to the international situation the number of available current Mathematical periodicals has diminished very considerably. The number of Journals received in exchange has come down from 69 to 36. No foreign journal published outside the United States of America and the British Empire is now being received. Even those still received, arrive irregularly and at much

longer intervals than before. The Librarian wishes to get all the old issues bound. Members are, therefore, requested to return all old issues that they can locate, and are warned that the circulation system may have to be suspended without further notice.

7. Regarding the rates of subscription for members the general body suggested to the Managing Committee at the last conference an examination of the life composition fees and the consideration in particular, of the proposal for a uniform rate of Rs. 10/- per annum for all members. The managing committee gave its best consideration to this proposal and resolved that the existing rates be not changed as the adoption of the uniform rate would adversely affect the finances of the Society.

The Managing Committee hopes that with the willing co-operation of our members, the financial position will improve by enlisting new members and institutional subscribers for the journals, and that more Universities, Provincial Governments and Ruling Princes will come to our aid by making handsome annual grants.

We are very glad to announce that Dr. A. Narasinga Rao of the Annamalai University has offered a prize for encouraging research in Mathematics to be awarded at our conferences. The conditions of award are being considered by the Managing Committee.

8. In conclusion, I take this opportunity of expressing our thanks to the Universities that have rendered financial help in the past by giving annual grants-in-aid, to the various local Governments, Indian States, Universities and other institutions for the facilities they have granted to their officers to enable them to attend this Conference.

I have also great pleasure in expressing, on behalf of the Managing Committee, our thanks to the Vice-Chancellor of the Aligarh University for giving us this opportunity of assembling here for this session.

Our Special thanks are due to Mr. S. M. Kerawala, his band of volunteers, and to the members of the Local Reception Committee for their warm hospitality and excellent arrangements for the comfort of the guests.

GLEANINGS

From childhood to old age Gauss always used the empty pages and the inside of the book covers as writing paper, not only for the entry of formulas often used and of occasional minor calculations, but frequently also for abstruse researches.

From G. W. DUNNINGTON'S
CARL FRIEDRICH GAUSS.

PRESIDENTIAL ADDRESS

BY

DR. R. VAIDYANATHASWAMY, *Madras University.*

Mr. Vice-Chancellor, Ladies and Gentlemen:

It is with great pleasure that I acknowledge the warmth of the welcome which has been accorded to this Mathematical Conference. When I look back on the previous conferences of the Indian Mathematical Society, it seems to me that the present conference will prove to be memorable in many ways. In the first place, in contrast to the peaceful atmosphere of our Mathematical deliberations in the past, we are meeting for the first time under the threat of impending war-conditions. The association between the tense exigency of the moment and the calm detachment of scientific thought is not perhaps so unnatural as it may appear at first sight; our temporal consciousness moves always between the two poles of the moment and the eternal, but these two poles even in their opposition seem to be linked in a mysterious kinship; for how else can we explain the fact that often in moments of intense stress, our minds get a sudden access to sensibility and becoming as it were, released from a prison, rise to a perception of universal things? We may recall that Archimedes, who with Newton and Gauss is reckoned among the three greatest Mathematicians of the world was in a state of concentration on a mathematical problem when the invaders of his city broke in upon him. Again, this conference is the first held by the Indian Mathematical Society in collaboration with a sister Mathematical Society; and this co-operation for a common cause is a happy augury for the future advancement of Mathematical research in this country. And lastly, this is the second time in succession that our Conference has been welcomed by a progressive University devoted to Muslim Culture—a culture that has made a respectable contribution to the early development of our Science.

The fact that in the past our Conferences have been enthusiastically received and welcomed by various Universities shows that, though Indian Universities are mainly engaged in the dissemination of knowledge, they are not blind to the fact that the primary function and justification of the University is research or the creation of new knowledge. It appears to me that the Universities in India are in a stage of transition at the present day. On the one hand, they are fettered by their past record of a mechanical reception of Western Knowledge and its mechanical transmission. Now, when the ideal of vernacularisation of all teaching is in the air, they are showing signs of discomfort and are beginning to be seized by stirrings towards a more creative and a fuller intellectual life. While we look forward to a post-war India with a revised political status, it will not I think be any undue optimism, to hope that Higher Education in this country will come to its own, and the Indian Universities will begin to function as the centres of higher knowledge and research and intellectual leadership in the arts and sciences.

It will certainly prepare the way for this inevitable transformation, if we realise that college instruction cannot be separated from research except to its detriment and degradation. The syllabus in Mathematics for a degree course, for instance, consists of a body of doctrines, together with the training for manipulating them in solving certain types of questions; these doctrines are advanced in text-books and by teachers, as if they had no background and no connection with any large human interests; as if they had always existed in their own right, because they were absolutely true and final—with that species of brutish finality which popular opinion finds it easier to attribute to Mathematics, than to sciences which are partly or wholly experimental. But this is a totally distorted view of the matter. The doctrines in our Mathematical syllabus represents the net gains of some two milleniums of intellectual effort of the human race; each particular idea or relationship was first manifested to an individual human intelligence after a long travail of seeking and the whole body of doctrine called Mathematics has been gathered and handed down as a precious relic, because of its inestimable value and significance for those ideals and for that fulfilment, for which the human race is striving through the ages. If this outlook towards Mathematical truth cannot be instinctively communicated by the teacher to his pupils, then what the teacher performs is not the teaching of Mathematics but its mechanisation and vulgarisation. And as a rule, the teacher will not be able to communicate this kind of feeling for Mathematics, unless he has himself gone through the great experience of Mathematical discovery. To the Seeker after Truth, this cardinal experience of discovery is a turning point, and may well be termed a baptismal experience, as it marks his birth into a world of deeper reality—the world of ideas. The experience of discovery is roughly the glimpsing of a new idea, by which the past knowledge is re-ordered and re-oriented, so as to exhibit hidden relationships and new and unsuspected values, so that there results a thorough transformation of outlook. A prosaic description of this kind does not really convey the sense and momentousness of this cardinal experience of the enlargement of human consciousness; for that, one must hark back to the Vedic and Puranic Symbolism. The Ushas of the Rig Veda, the Deity of the Dawn is nothing else but this experience, and the glamour of this Deity, and the number of the hymns addressed to her and their fervid passion, show how deeply the Vedic Rishis had realised and experienced the recurrent enlargements of the aspiring evolving human consciousness. Similarly the glamourous figure of Lakshmi rising from the ocean of milk churned by the Devas and Asuras is the pre-eminent symbol of something absolutely new and original, something fresh from the primordial mint of creation, rising in the human consciousness and miraculously transforming the whole outlook and revealing a new world of values. It is only that person who has experienced the great cosmic mystery of the rising of the original Lakshmi from the churned Ocean of Milk that can penetrate behind the Symbolism of the cheap vulgarisations of the figure of Lakshmi, which can be bought in the bazaars; in a similar manner, it is only the teachers who have gone through the baptismal experience of Mathematical discovery, that can convey to his pupils the inner

significance of the text-book Mathematical doctrines, relive with them the original glamour of their first discovery, their first revelation to a thrilled human mind, and communicate a feeling of their place and value in the journey of the human race, the Epic of Civilisation.

The Indian student or Indian Teacher of the present day wishing to undertake Mathematical research, is confronted with the vast Mathematical material which is the legacy of the 19th century. The 19th century has been well called the Golden Age of Mathematics, its achievements belittling that of all previous centuries taken together. The developments of this century in "Analysis" and "Function-Theory", in Algebra and Geometry, in Arithmetic and in Invariant-Theory, in Mechanics, have so changed the face of Mathematics as to alter it beyond recognition and have won for it a far-flung dominion. Indeed some of the things which are a result of the work of this century, and which we teach young pupils of the College classes belong to a conceptual level that would baffle the understanding of the mighty intellects of antiquity or of the Middle Ages. The basic ideas of the 19th century work do form or should form part of the course in our Honours and post-graduate classes. The Mathematical aspirant who wishes to push on to research, should of course discipline himself thoroughly in the methods of great masters of the 19th century; this would not be difficult to accomplish in any particular topic, though the extent and refractory character of the material would make it difficult to do in several topics at once. The worker would then be naturally led on to extend the scope of these methods or to complete the solution of some problem by taking it up at the point where it had been left off. This would be the usual way which workers might be expected to take and do actually take both here and in other countries. But is this the right way? I think not. Let me explain why. Some young Indians inspired and fascinated by Euclid's Elements, are enticed to further research and do succeed in finding ingenious proofs of difficult riders or in constructing elegant theories within the ambit of Euclid's method and view-point. Others, of a more skeptical or valorous disposition attempt a proof of the parallel postulate by the classical methods of Euclid; these are by no means cranks or irrational persons; for if you limit yourself to the view point of Euclid, it is not at all certain that the parallel postulate could not be proved. Euclid, of course, guessed that it could not be, but it was only a guess, and he might have well been wrong. It was only in later times and from a wider point of view that it became possible to prove that the parallel postulate cannot be proved. My point is that in neither of these cases is there any really creative work, because there is no advance beyond the point of view of Euclid — — the point of view of two milleniums ago. On the other hand what the true nature of creative work in geometry should be is well demonstrated by the actual historical course of development. The transformation of Euclidean Geometry into Geometry as we think of it today was effected by the impact of some half-a-dozen new ideas in succession on the material of Euclidean Geometry. Each of these impacts irradiated the material and raised it to a higher plane of value and significance. The Treatise on the Conic

Sections of the Greek Geometer Appolonius was *not* one of these impacts, because though an extension and addition to Euclid's work it was not a *creative* addition in the true sense, it belonged to the same value-level and the same significance-level as Euclid's elements. These impacts may be easily enumerated; the first was Descartes' discovery of Algebraic Geometry, the second was the principle of continuity associated therewith, which brought in imaginary elements into geometry and the concept of *n*-dimensional geometry. The next was the discovery of non-Euclidean Geometry with the conviction of its *reality*. The next was the discovery of projective geometry with the importing of infinite elements. The next was the concept of group and invariant culminating in Klein's grand generalisation of geometry as the invariant theory of a transformation group. The last was Gauss' discovery of differential geometry and Riemann's concept of curvature of a manifold. Each of these new ideas illuminates the subject matter in such a way that it might well be called a "DAWN" in accordance with the Vedic imagery; and this name is all the more appropriate because the illumination in each case has come in response to a seeking for light in order to clear up some felt darkness in the subject. Thus in brief Euclid's conception of geometry and ours are separated by some half a dozen dawns.

This being the nature of creative research, the present day Mathematical aspirant should seriously ask himself whether in dealing with the rich legacy of the 19th century, he is really bound to limit himself to its view point and method, whether there have not happened subsequent dawns, dawns of this century, the illumination of which has yet to play on the older material transforming it and raising it to its own level of value and significance. If there have been such dawns, the path of creative research lies most definitely in the fulfilment of the Time-Spirit by assimilating the older material to the newer light.

We have today arrived at a deeper insight into the nature of Mathematical thought than was ever possible before. As the culmination of the researches in the analysis of mathematical reasoning initiated in the last century, we realise today that Mathematics should be conceived as 'the class of all deductive systems' or as 'the analysis of structure'. This is no doubt a highly abstract view, but abstraction where Mathematics is concerned, is not a flight from the concrete, but a temporary stepping back from the subject matter, in order to get it into clearer perspective and return upon it with greater power and insight. Looking back on the past, we may recognise that this intuition of the nature of Mathematics has been always present from the earliest times as far back as Euclid, though in a germinal form without rising to self-awareness.

Another curious feature of this modern view of Mathematics is the laying bare of the peculiar relation which holds between Mathematics and Logic—a relation, not of one-sided dependence as was thought, but of two competitors in a race each of whom tries to outstrip the other. For, it turns out that logic—or rather, any particular level or system of logic—has a structure of its own and

can be formalised and presented as a deductive system, in which the processes and reasonings depending on the actual 'content' as opposed to the 'form' of the system belong to a deeper unformalised level of 'meta-logic.' This must necessarily turn out to be the case with a human mentality capable of deeper and ever deeper levels of reflection on its own processes.

The 'Structures' or 'Deductive Systems' of Mathematics may be of a very general or even of an arbitrary character. But in practice, the purposive character of Mathematical science brings in some limitations. Historically Mathematics had the task of elucidating the problems of practical life. Its central and distinctive contribution in this regard is the concept of the 'real number,' by which the sense world can be 'measured,' and its manifold relations explained. This concept runs like a spinal cord through the whole of Mathematics, furnishing a solid support to its abstract structures and a centre of reference to its boundless peregrinations Indeed, when any general Mathematical theory or result has to be brought home by an illustration or to be used or applied, we have to bring in the real number directly or indirectly. It will not be unnatural then to except the typical Mathematical Structures to have a close bearing on the real number concept, to hover round it, as it were, or even to have been implicitly involved in its development.

The concept of the 'real number' has developed through four stages, the cardinal number, the natural number, the rational (or fractional number), and the real number, and we do find Mathematical Structures typical of each of these stages of its evolution. The 'Cardinal number' is the signless integer used in counting, and the addition of these cardinal numbers must be explained in terms of the union of sets without common elements. Thus the notion of cardinal number must be logically explained in terms of set-algebra. But the notion of set has a definite content, and is therefore 'concrete' and not 'abstract' according to the standards set by Mathematics. To get at the formal characteristic of set-algebra, we observe that both set-union and set-intersection can be explained in terms of the relation of set-inclusion. Therefore the abstract Mathematical Structure revealed in set-algebra and in the notion of cardinal number is the 'partially ordered set,' namely a set of elements constituting the field of a binary-reflexive transitive relation, formally analogous to set-inclusion. It is not surprising that the structural idea of the 'partially ordered set' includes logic also among its applications, in view of its close connection with the fundamental logical notion of 'set.'

The cardinal numbers are not closed for subtraction, and the impulse which conceives the 'natural numbers' or the set of positive and negative integers, is clearly the impulse of 'completing the group.' Thus the Structural idea which is manifested at the second stage of evolution is the idea of 'group.' The passage to the rational number is similarly effected by completing the multiplicative group of the natural numbers other than zero; this brings forth the structural idea of 'field' which is or rather ought to be, the subject-matter of Algebra. In the final stage, the notion of real number is arrived at from

the rational number by a *coup d'etat* accepting an infinite process. According to Dedekind, the motivation in this process is to arrive at an analysis and a logical explanation of the geometrical intuition of 'Continuity.' The Mathematical discipline which analyses and investigates the structure of the notion of continuity is called 'Topology.' Accordingly the structural moment revealed in the passage from the rational to the real number must be called the 'topological moment.' Thus corresponding to the four stages of evolution of the real number, we have four principal structure-types, the 'partially ordered set' and 'topology' at the extremes, and the intervening structural levels of 'group' and 'field' which are usually included under 'Algebra.' These are of course 'elementary' structure forms, and may be found mixed up together in the background of any actual piece of mathematical reasoning. In order to study these structures effectively, Mathematics would also have to deal with generalised forms of a looser structure, as for instance 'Semi-group' or 'ring.' Also, an infinite variety of secondary or derived structure-types may arise, based on these; as for instance in Geometry, where we have to deal with the structure of sets of linear equations in a field. Thus, in fine, we have to ascribe a central position in Mathematics, to the four structural moments revealed in the evolution of the real number, without prejudice to the fact that Mathematics is concerned with deductive theories based on any system of postulates whatever.

Every discovery of new and far-reaching ideas must be followed up by a period in which their scope and applicability to the existing material is surveyed and worked out. Such work is necessary not only for an increased insight into the existing body of knowledge, but also for giving content to the new ideas and fixing their bearings. It is work in this direction that is called for at the present moment, and I feel strongly that Indian Mathematicians and workers should orient themselves accordingly, so as not to be behindhand in doing their share.

GLEANING

Few people realize how much the *geometric ideas of symmetry*, congruence, equality, and similarity influence their lives. The autombile in which we ride has many applications of these principles. The stamping dies press into form sections of the body and many of its fittings. These are produced as congruent parts by simple repetition. If this were not true, a car would cost several times its present price. The difference in the cost of tailor-made and factory-made suits is the result of the application of congruence and similarity. Many congruent portions are made at a single cutting by machinery. Sizes 30 to 49 exemplifies similarity, the suit as a whole represents symmetry.

A. B. Miller in *The Mathematics Teacher* Dec. 1941.

ABSTRACTS OF PAPERS

P. Kesava Menon, Annamalai University.

A Test for Groups of Primes.

In a paper entitled 'A test for groups of primes'* Dr. S. S. Pillai has obtained the following results:—

(i) the necessary and sufficient condition that both p and $p+r$ be primes is

$$(p-1)!+1+\frac{p(r!-1)}{(r\cdot r!)}\equiv 0 \bmod p(p+r),$$

and

(ii) the necessary and sufficient condition that the three numbers p, $p+r$, $p+s$ be primes is

$$(p-1)!\ +1+\frac{p(r!-1)}{r\cdot r!}+p(p+r)\left\{\left(\frac{r!-1}{r\cdot r!}-\frac{s!-1}{s\cdot s!}\right)\Big/(s-r)\right\}$$
$$\equiv 0 \bmod p\ (p+r)\ (p+s).$$

He further states that these results may be extended to groups of larger number of primes but that he has not been able to reduce the conditions for the general case to any simple form. In this paper I prove the following general result:

The necessary and sufficient condition that

$$p+r_0,\ p+r_1,\ .\ .\ .\ .\ ,\ p+r_n\ \ (r_0=0)$$

be primes is

$$(p-1)!\ +1+\sum_{i=0}^{n-1}(-1)^{i+1}\,a_{i+1}\,(p+r_0)(p+r_1)\ .\ .\ .\ (p+r_i)\equiv 0$$
$$\bmod p(p+r_1)\ .\ .\ .\ (p+r_n),$$

where

$$a_i=\sum_{j=0}^{i}\frac{1}{r_j!\,\phi_i'(r_j)},\quad \phi_i(x)=(x-r_0)(x-r_1)\ .\ .\ .\ (x-r_i).$$

D. R. Kaprekar, Devlali.

On Superwonderful Demlo Numbers

Numbers of the form

$$W_p=123456789_{(p)}87654321$$

where $9_{(p)}$ stands for the digit 9 repeated p times are considered in

* *Journal of the Indian Mathematical Society* Vol. XVII (1927).

this paper. With the help of a table of recurring periods of M/81 for different values of M from 1 to 80, rules are given for writing down the product of a W_p with any integer with little effort. Such products are sometimes very striking. Thus

$$W_3 \times 1377 = 1699999998283000000017$$

S. S. Pillai, Travancore University.

1. *On the number of numbers which contain a fixed number of factors*

Let (1) $\sigma_\nu(x)$ denote the number of numbers $\leqslant x$, which are composed of ν prime factors, where each factor is counted according to its multiplicity; (2) $\pi_\nu(x)$ denote the number of such square-free numbers, and (3) $\rho_\nu(x)$ denote the number of numbers $\leqslant x$, which are composed of ν *different* prime factors.

Then when ν is fixed, Landau has proved in '*Primzahlen*' (pp. 202—213), that

$$\sigma_\nu(x) \sim \pi_\nu(x) \sim \rho_\nu(x) \sim \frac{x}{\log x} \cdot \frac{(\log\log x)^{\nu-1}}{(\nu-1)!}.$$

When ν is a function of x, nothing is known.

In this paper, I prove that the above asymptotic formulae hold good when $\nu = O(\log\log x)$; further I show that if x is sufficiently large,

(i) when $\nu \leqslant k \log\log x$, there is a constant $H = H(k)$ such that

$$\pi_\nu(x) \geqslant H \cdot \frac{x}{\log x} \cdot \frac{(\log\log x)^{\nu-1}}{(\nu-1)!};$$

(ii) when $\nu \leqslant 1{\cdot}015(\log\log x - 1{\cdot}7)$,

$$\sigma_\nu(x) \geqslant \frac{5}{27} \cdot \frac{x}{\log x} \cdot \frac{(\log\log x)^{\nu-1}}{\nu-1!},$$

and

(iii) when $\nu \leqslant 1{\cdot}00035(\log\log x - 3{\cdot}2)$,

$$\pi_\nu(x) \geqslant \frac{1}{32} \cdot \frac{x}{\log x} \cdot \frac{(\log\log x)^{\nu-1}}{(\nu-1)!}.$$

In the body of the paper other results will be found.

There is reason to guess that the asymptotic formulae hold good for $\sigma_\nu(x)$ where $\nu \sim \log\log x$. It is intensely interesting and at the same time terribly difficult to find out the critical value of ν when

the asymptotic formula breaks down. Even a guess seems to be impossible in the present state of knowledge.

2. *On a lattice point Problem*

Let $f(x) = x - [x] - \frac{1}{2}$ when x is not an integer and $f(x) = 0$ when x is an integer; and let $F(\theta, n) = \sum_{r=1}^{n} f(r\theta)$. The paper discusses the properties of $F(\theta, n)$.

Hansraj Gupta, Govt. College, Hoshiarpur

On Numbers of the form $4^a(8b+7)$

All numbers which cannot be expressed as the sums of less than four squares are of the form $4^a(8b+7)$ where a and b are non-negative integers. If $K(n)$ denotes the number of such integers not exceeding n, this paper is concerned with an expression for $K(n)$ and a method of calculating $K(n)$. The results are an improvement on those of Mr. S. C. Chakrabarti in *Bull. Calcutta Math. Soc.* 32 (1940).

The paper also deals with the number of solutions of the equations

$$n - 6K(n) = m, \quad 0 \leqslant n < 4^s$$

and

$$n - 6K(n) = m, \quad 4^s \leqslant n < 4^{s+1}, \quad s \geqslant 1.$$

M. Ziaud Din, Lahore.

On partitions and divisors of a number

Symmetric functions and determinants have been applied to derive certain formulae in partitions and divisors of an integer.

F. C. Auluck, Lahore.

On the maximum value of the number of partitions of n into k parts

M. Venkatarama Iyer and M. Bhimasena Rao, Bangalore.

On types of solutions of $x^3+y^3+z^3=1$ *in integers*

K. Sambasiva Rao, Andhra University, Waltair

On the function $\gamma(k)$ of E. M. Wright

We call a solution in positive integers of

$$x_1^k + x_2^k + \cdot \cdot \cdot + x_m^k = y_1^k + \cdot \cdot \cdot + y_n^k \tag{1}$$

non-trivial if

(A) no x is equal to any y,

and (B) $(x_1, \cdot \cdot \cdot \cdot, x_m, y_1, \cdot \cdot \cdot \cdot y_n) = 1.$

E. M. Wright* denotes by $\gamma(k)$ the least value of n such that (1) has infinitely many non-trivial solutions with $m < n$ and proves that

$$\gamma(k) = O\{(1{\cdot}307)^k\}.$$

S. Chowla† defines $\gamma(k)$ as above but without the restriction (A) and proves that

$$\gamma(k) \leqslant \tfrac{1}{2}(k^2 + k) + 1.$$

In this paper the author considers the functions $\gamma(k)$ of Wright, and proves that

$$\overline{\lim_{k \to \infty}} \frac{\gamma(k)}{k \log k} \leqslant 2.$$

K. Chandrasekharan, Madras University.

On the (J_μ, λ) summation of Series

In an unpublished memoir, Dr. S. Minakshisundaram has made use of a new method of summation which is defined as follows :—

A Series $\sum a_n$ is said to be summable (J_μ^k, λ) to the sum S, if

(i) $\phi_\mu^k(t) = \sum\limits_{n=0}^{\infty} a_n \, \alpha_\mu^k(\lambda_n t)$ is convergent for real $t > 0$,

(ii) $\phi_\mu^k(t) \to S$ as $t \to 0$,

where k is a positive integer, (λ_n) is an increasing divergent sequence of positive numbers, and $\alpha_\mu(x) = 2^\mu \Gamma(\mu + 1) \dfrac{J_\mu(x)}{x^\mu}$, $J_\mu(x)$ denoting the Bessel function of order μ.

* E. M. Wright: "On Sums of kth powers," J. L. M. S., Vol. 10 (1935), pp. 94-99.

† S. Chowla: "A Theorem on Sums of Powers with applications to the additive theory of Numbers." *Proc. Ind. Acad. Sci.* Vol. I (1936) pp. 701-6.

The object of this paper is to prove the following results expressing the relationship between the above process and Cesàro's or Riesz's process as the case may be :—

I. If $\sum a_n$ is summable (Cr), then it is summable (J_μ^k, n) for $\mu > \frac{r+1}{k} - \frac{1}{2}$

II. If (i) the series $\sum a_n \, \alpha_\mu (\lambda_n t)$ converges for all $t > 0$, and uniformly in every finite interval,

(ii) $\phi_\mu (t)$ is bounded as $t \to \infty$,

(iii) $\phi_\mu (t) \to s \neq \infty$ as $t \to 0$,

then $\sum a_n$ is summable $(R;\, \lambda, k)$ for $k > \mu + \frac{1}{2}$.

V. S. Krishnan, Madras University.

Homomorphisms between distributive Lattices

Homomorphisms between lattices have not so far been considered in any detail. The purpose of my paper is to study general homomorphisms between distributive lattices—which preserve finite sums and products, and the special cases of these, called by me 'Regular,' which are determined solely by ideals in the first lattice. By establishing a regular homomorphism between a lattice L, in which every element has a product complement, and this set N of normal elements of L, it is possible to study the normal elements from an entirely new point of view.

P. D. Shukla, Lucknow.

On the differentiability of the integral.

The necessary and sufficient condition for the existence of $F'(0)$, where

$$F(x) = \int_0^x \phi\, [\psi(x)]\, dx,$$

and where $\psi(x) \to \infty$ as $x \to 0$, and $\phi[\psi]$ is any finite and continuous function of ψ having a discontinuity of the second kind at $x=0$, has been investigated. Under certain restrictions on the function $\phi[\psi(x)]$, this problem has been studied earlier by the author (*Proc. Ind. Sc. Congress, Benares* (1941), Pt. III, pp. 6-7). In the present paper those restrictions have been removed and the necessary and sufficient condition for the existence of $F'(0)$ has been found out in the most

general case of ϕ. An attempt has also been made in the present paper to show that the results about $F'(0)$ as found out in a special case of ϕ by G. Prasad are the same as those found by the author in the general case of ϕ.

S. M. Shah, Muslim University, Aligarh.

1. *On the lower order of an integral function.*

I define the lower exponent of convergence of zeros of $f(z)$ and prove two theorems.

THEOREM 1. *Given $\rho\,(0<\rho<\infty)$ there exists an integral function $f(z)$ of order ρ for which $\rho=\rho_1$ but for which $\lambda>\lambda_1$*

Here ρ_1 is the exponent of convergence of the zeros and λ the lower order of $f(z)$.

$$\lambda_1 = \underline{\lim}_{r=\infty} \frac{\log n(r)}{\log r}$$

is the lower exponent of convergence of zeros of $f(z)$.

THEOREM 2. *If $f(z)$ be of order ρ, $0<\rho<1$ then $\lambda \leqslant \dfrac{\lambda_1}{\lambda_1+1-\rho}$*

2. *Note on a Theorem of Polya.*

Let $f(z)$ be an integral function of order ρ G. Polya proved that

$$\underline{\lim}_{r\to\infty} \frac{n(r)}{\log M(r)} \leqslant \rho \tag{1}$$

where $M(r)$ and $n(r)$ have their usual meanings. Valiron showed that if $f(z)$ be of finite order ρ.

$$\underline{\lim}_{r\to\infty} \frac{n(r)}{\log M(r)} \frac{\log r}{\log \log M(r)} \leqslant 1 \tag{2}$$

In this note I prove a better result, namely.

If $f(z)$ be an integral function, and if

$$\lambda_1 = \underline{\lim}_{r\to\infty} \frac{\log n(r)}{\log r}$$

then

$$\underline{\lim}_{r\to\infty} \frac{n(r)}{\log M(r)} \leqslant \lambda_1. \tag{3}$$

V. Ganapathy Iyer, Annamalai Unversity

A property of the maximum modulus of integral functions

Let E be a measurable set in $(0, \infty)$. Let $E(r)$ denote the measure of E in $(0, r)$. Then

$$\overline{\lim_{r\to\infty}} \frac{E(r)}{r} \text{ and } \underline{\lim_{r\to\infty}} \frac{E(r)}{r}$$

are called the upper and lower densities of E.

Now let $f(z)$ be an integral function and $M(r, f) = \max_{|z| \leq r} |f(z)|$. The order ρ is defined by

$$\overline{\lim_{r\to\infty}} \frac{\log \log M(r)}{\log r} = \rho.$$

Let E be the set of points in $(0, \infty)$ such that

$$\lim \frac{\log \log M(r)}{\log r} = \rho$$

as $r \to \infty$ over E. I show that E has upper density one and that it is the best possible result in the sense that there are functions for which the lower density of E is zero. I prove a similar result for functions of order zero but of finite logarithmic order. I apply these results to study the precise order of composite integral functions.

R. S. Varma, Lucknow University.

Some infinite series involving Humbert Functions

In this paper I have investigated some infinite series involving Humbert Functions. The product of two Humbert Functions is expressed in terms of generalised Hypergeometric functions. The problem of convergence is discussed with the help of the asymptotic behaviour of the function recently obtained by me in a paper published in the April 1941 issue of *Annals of Mathematics.*

P. C. Mital, Lucknow University.

Operational images of functions which are self-reciprocal in the Hankel Transform.

In this paper, I have given two theorems for investigating the operational image of a function which is self-reciprocal in the Hankel Transform.

Hari Shanker, Anglo-Arabic College, Delhi.

On Hankel Transforms of the product of two parabolic cylinder functions.

The object of this paper is to establish the relation

$$\int_0^\infty D_\mu (ix)\, D_{-\mu-2n-1} (x)\, x^\nu\, J_\nu (ax)\, dx$$

$$= \frac{2^\mu\, \mu!\, \Gamma(2\nu+1)\, 2^{(n-3\nu-1)/2}\, a^{n-1}\, e^{\frac14 a^2}}{\Gamma(\nu+1)} \times$$

$$\sum_{r=0}^{\mu} \frac{\Gamma(n+r+\frac12)(-\sqrt{2}a)^r\, W_{-\frac{(n+\nu+r)}{2},\, \frac{n-\nu+r}{2}} (\frac12 a^2)}{(\mu-r)!\; r!\; \Gamma(2n+r+1)}$$

and prove the following Theorem:—

If $\quad \phi (x) = x^{\nu-\frac12}\, D_\mu (ix)\, D_{-\mu-2n-1} (x)$

and $\quad \psi (x) = \dfrac{i^\mu\, n!\, \Gamma(2\nu+1)\, x^{n-\frac12}\, e^{\frac14 x^2}}{\Gamma(\nu+1)\, 2^{\frac{3\nu-n+1}{2}}} \times$

$$\sum_{r=0}^{\mu} \frac{\Gamma(n+r+\frac12)(-\sqrt{2}x)^r\, W_{-\frac{(n+\nu+r)}{2}\, \frac{(n-\nu+r)}{2}} (\frac12 x^2)}{(\mu-r)!\; r!\; \Gamma(2n+r+1)},$$

then $\phi(x)$ and $\psi(x)$ are Hankel Transforms of each other provided that μ is a positive integer, $2n+1>0$, $R(2\nu+1)>0$ and $R(2n-\nu-\frac32)>0$.

G. S. Diwan, Bombay.

The "Unique Particular Integral" in the theory of ordinary linear differential equations.

In this paper are discussed the difficulties in fixing on any particular solution of $f(D)\, y = X$ as the "unique particular integral" and the advantages resulting therefrom in the treatment of specific problems. The purposes for which this conception of uniqueness are introduced are clarity in reasoning, and simplification of the proofs of certain propositions. Examples are given to show how these objects are achieved.

S. Minakshisundaram, Madras University.

On Expansion in Eigenfunctions

Let $\omega_n(x, y)$ be the normal orthogonal eigenfunctions, with the corresponding eigen-values $\lambda_n > 0$, $n = 1, 2, \ldots$ arranged in increasing order of magnitude, of the boundary value problem

$$\Delta\omega + \lambda^2\omega = \frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} + \lambda^2\omega = 0 \qquad \ldots \quad (1)$$

$$\omega\,(x, y) = 0 \text{ on C} \qquad \ldots \quad (2)$$

C being the boundary of a bounded regular domain D.

If $f(x, y)$ is an arbitrary function whose square is integrable L in D and which vanishes on C, and if

$$f(x, y) \backsim \sum a_n\,\omega_n\,(x, y) \qquad \ldots \quad (3)$$

$$a_n = \iint\limits_{D} f\,\omega_n\,dx\,dy \qquad \ldots \quad (4)$$

the relationship between the behaviour of $f(x, y)$ in the neighbourhood of an interior point of D and summability by typical means of the series on the right of (3) is studied.

As a particular instance it might be mentioned that:

At a point of continuity of $f(x, y)$, $\sum a_n\,\omega_n$ *is summable* (λ, k), $k > 3/2$, *to the value of the function and at a point where* $\sum a_n\,\omega_n$ *is convergent*

$$\lim_{r\to 0} f_r\,(x, y) = \lim_{r\to 0} \frac{1}{\pi r^2} \int_0^r \int_0^{2\pi} f(x + \rho\cos\theta, y + \rho\sin\theta)\,\rho\,d\rho\,d\theta$$

exists and is equal to the sum of the series.

Prithvi Nath Sharma, Hindu College, Delhi.

On a hystero-differential equation

The general solution of the hystero-differential equation $f^n(x) = f(1/x)$ is obtained in this paper. Its particular case of $n = 1$ has been recently discussed by Silberstein (*Phil. Mag.*, 30 (1940), 185-7).

Santi Ram Mukerjee, Allahabad.

On the equation $f^r(x) = f(\pm 1/x)$

The solutions corresponding to $r=1$ to $r=6$ have been studied as also the behaviour of the solution in the general case.

A. A. Krishnaswami Ayyangar and W. F. Beard, Mysore.

A genetic problem in geometry

For some time, the authors have been tackling a genetic problem in geometry connected with the following well known result:

ABC is a triangle with O as its circumcentre. PQR is a triangle inscribed in ABC so that O is its orthocentre. Then, this triangle has one of two dissimilar properties:

(i) it is self-polar w. r. t. the circumcircle of ABC;

(ii) it is similar to ABC.

The existence of the alternatives is easily accounted for by a certain hyperbola which Mr. Beard has discovered geometrically. But why these twins should differ so radically from each other in their geometrical character, is not yet satisfactorily explained.

The following general problem is solved:

Given two points O, P and two fixed straight lines AB, AC, to find two other points Q, R on AB, AC respectively such that PQR may have O as its orthocentre.

Incidentally it is noted that if the base of a triangle is fixed and the vertex describes a curve of the n-th degree, the locus of the ortho-centre is, in general, a curve of the $2n$-th degree, of which the extremities of the base and the point at infinity in the direction perpendicular to the base are n-ple points.

S. M. Kerawala, Muslim University, Aligarh

1. *On the Frégier Forms associated with Conics*

The aim of this paper is to probe into the full implications of the well-known theorems of Frégier in the theory of conic sections. A number of interesting loci and envelopes are thus obtained, and

their intimate relationship with the theory of inversely similar figures is studied. In the light of these investigations the porism of Poncelet is finally examined for particular cases.

2. *On a Transformation for duplicating the condition of Poncelet's Porism*

The two conics of the problem are assumed to have been reduced by a homographic transformation to two circles of radii r and R with their centres at a distance d apart. Chaundy has proved that in order to convert the porism $P(2n+1)$ into $P(4n+2)$ of the separable case, it is sufficient to interchange R and d. In this paper, it has been proved that any $P(n)$ is converted into $P(2n)$ if in $P(n)$ be substituted $(R^2-d^2)^2$ instead of R, $2r^2(R^2+d^2)-(R^2-d^2)^2$ instead of r and $4r^2 Rd$ instead of d.

K. Rangaswami Aiyar, Annamalainagar.

On the space representation of Hart Tetrads.

We represent the circles of a plane by the points of a projective [3], so that the point-circles are represented by the points on a quadric Q, the same symbols denoting circles of the plane and also their representative points in [3].

Let *abcd* be a Hart tetrad δ, $A_1B_1C_1D_1$ its complementary Hart tetrad and ABCD, the polar tetrad Δ of δ with respect to Q. Also let $\bar{Q}$ be the harmonically conjugate quadric of Q with respect to Δ H the unique quadric in the pencil determined by Q and $\bar{Q}$ circumscribed to Δ, Δ_1 the unique tetrad inscribed in H and desmic to Δ and Δ_2 the tetrad which with Δ, Δ_1 forms a desmic system. Further let us denote by Γ_1 the desmic transformation whose singular points are the vertices of Δ_1 and fixed points the vertices of Δ, Δ_2.

It is known that $\bar{Q}$ is a pair of planes intersecting in a line l and also that a, b, c, d, A_1, B_1, C_1, D_1 lie on a twisted cubic L.

We prove that: (i) A_1 B_1 C_1 D_1 are the vertices of Δ_1 and (ii) the transform of l in the cubic transformation Γ_1 is the twisted cubic L. (i) is true even when A_1, B_1, C_1, D_1 are the circles which cut a, b, c, d *all* at the same angle.

C. N. Srinivasiengar, Engineering College, Bangalore

On the Quartic Developable

It is shown that the first polar surface of any point on the twisted cubic forming the edge of regression of the developable is a ruled cubic of Cayley's type. The family of Cayley's cubic scrolls obtained as the first polars of various points on the twisted cubic possesses some interesting properties which are worked out in this paper. The nature of the first polars of other points on the developable is also investigated.

V. Rangachariar, Science College, Patna

On Systems of Quadrics in a [4]

Three quadrics in a [4] intersect in an octavic curve, through which a doubly infinite system—a net—of quadrics can be drawn. Each quadric is made to correspond to a point in a plane σ. The cones of the net correspond to a certain quintic curve without double points. The locus of the vertices of these cones is a curve. Most of the properties related to the corresponding three dimensional problem in W. L. Edge's paper (*Proc. Lond. Math. Soc.* 1931) is extended to the present case. The properties of a set of quadrics consisting of a triply infinite system through 16 associated points is also discussed.

Ram Behari, St. Stephen's College, Delhi

A Property of the Principal Ruled Surfaces through a line of a Normal Rectilinear Congruence

In a previous paper (published in the *Proc. Ind. Acad. Sc.* Vol. XII, 1940, pp. 205-207) I proved analytically the theorem that the curves on the sphere representing the distributive ruled surfaces through a line of a normal rectilinear congruence are isometric The object of this paper is to show analytically that the same result is true for the principal ruled surfaces also.

A. Narasinga Rao, Annamalai University.

On an unusual metric defined by line elements in the inversive plane.

Two line elements in the plane of inversion define an invariant angle, namely that at which the 2 circles one through each element

and the point of the other element cut each other. Some of the peculiar features of this invariant of inversive geometry are studied here. Essentially it is the study of an invariant defined by 2 tangent lines to a quadric in a projective S_3 under the group of projective transformations carrying the quadric into itself.

Mohammad Shabbar, Aligarh

Continuous groups of affine collineations and local applicabilities and their applications to Lorentz-invariant Riemann Spaces.

I study here the groups of affine collineations and local applicabilities in Riemannian spaces admitting the Lorentz-group L_6 whose generators are:

$$x^2 \frac{\partial}{\partial x^1} + x^1 \frac{\partial}{\partial x^2};\ x^3 \frac{\partial}{\partial x^1} + x^1 \frac{\partial}{\partial x^3};\ x^4 \frac{\partial}{\partial x^1} + x^1 \frac{\partial}{\partial x^4};$$

$$x^3 \frac{\partial}{\partial x^2} - x^2 \frac{\partial}{\partial x^3};\ x^4 \frac{\partial}{\partial x^3} - x^3 \frac{\partial}{\partial x^4};\ x^4 \frac{\partial}{\partial x^2} - x^2 \frac{\partial}{\partial x^4}.$$

The spaces have been classified according to the groups they admit and the properties of these groups have been studied.

R. Vaidyanathaswamy, Madras University.

Related topologies.

K. Suryanarayanamurthi, Madras University

On the problem of Metrisation in neighbourhood spaces

The object of this paper is to examine the equivalence of the two sets of axioms—Hausdorff axioms and Kurutowski axioms, for a Topological space and analyse in particular the conditions for metrisations of Neighbourhood spaces. The following results are proved:—

I. A T_1—space satisfying the first countability axiom is metrisable if and only if it is completely regular.*

II. A T_1—space with equi-continuous functions satisfies the first countability axiom.†

* A Space is completely regular if, when P and Q are any two closed sets in it of which P is a single point set (p), there exists a continuous function $f(p)$ such that $f(p)$ is equal to 0 for $P \in (p)$, $f(p) = 1$ for $p \in Q$ and $0 \leq f(p) \leq 1$ throughout.

† If given an ε there is a neighbourhood U such that the oscillation of each function f of the family F in U is less than ε, then all those functions of F are said to be equi-continuous.

By the second result it is shown that the theorem of metrisation of Neighbourhood Spaces, due to Chittenden ‡ in terms of equi-continuous functions is the same as the result I.

C. N. Srinivasiengar, Engineering College, Bangalore.

The Resultant of wrenches on two given screws

The cylindroid associated with a pair of given screws of arbitrary position is worked by considerations which are more direct than those in the familiar method of Ball's *Theory of Screws.* A number of new results regarding the properties of the cylindroid are also worked out.

B. R. Seth, Hindu College, Delhi.

Transverse vibrations of triangular membranes.

Besides the known solutions of a right-angled triangle and that of an equilateral triangle new results have been obtained for

(i) an isosceles triangle containing an angle of 120°;

(ii) a right angled triangle containing an angle of 60°.

The frequencies in the known cases are given by

$$\frac{p}{2\pi} = \frac{c}{4a}\left[(2m+1)^2+(2n+1)^2\right]^{\frac{1}{2}}, \qquad \frac{p}{2\pi} = \frac{c}{a}\left[m^2+n^2+mn\right]^{\frac{1}{2}}.$$

In the case of (i) and (ii) it is found that

$$\frac{p}{2\pi} = \frac{c}{2a}\left[(2m+1)^2+(2n+1)^2+(2m+1)(2n+1)\right]^{\frac{1}{2}},$$

where c and a are known constants, and m and n are integers.

K. Nagabhushanam, Andhra University, Waltair.

Subgroups of the infinitesimal Contact Transformations

The following subgroups of the general infinitesimal contact transformations of $p_r\, dq^r - \mathrm{H}dt$, are considered:

(i) those confined to a surface which is an integral of Hamilton's equations, such as surfaces of constant energy;

‡ *Bulletin of the American Mathematical Society* Vol. 33; 1927.

(ii) those displacing points on a trajectory into a neighbouring one on the same trajectory;

(iii) those for which $dW = 0$, where

$$p_r\, dq^r - H dt = (p')_r\, d(q')^r - H'\, dt' + dW.$$

From (i) and (iii), further subgroups can be derived; but (ii) and (iii) mutually exclusive, and no further subgroups are obtainable.

Ram Ballabh, Lucknow University.

Superposable Fluid Motions

The paper deals with superposable fluid motions of which the vortex lines coincide with stream lines. It has been established that if λ, the proportionality factor between the vorticity and velocity components, be a constant, the stream lines of the motions can lie on concentric spheres, coaxial cylinders and parallel planes but *not* on coaxial planes or coaxial cones having a common vertex. Other results have been obtained regarding λ being *a function of space* variables and t.

G. R. Seth, Hindu College, Delhi.

Motion of a parabolic cylinder through a viscous liquid.

The resistance suffered by a parabolic cylinder in a non-viscous liquid of density ρ is $\pi \rho a U^2$ where U is the uniform velocity and $4a$ is its latus-rectum, provided the pressure at infinity is taken to be zero.

When the liquid is viscous and we neglect the inertia terms, we get a solution of the problem in which the velocity at infinity becomes as great as $\sqrt{r}$. In the corresponding solution of a circular cylinder the velocity at infinity varies as $\log r$. But we do not get a finite value for the drag.

On applying Oseen's approximation to the problem all the boundary conditions, including the one at infinity, are satisfied, but again no greater value of the drag is obtained.

S. R. Das, Delhi.

1. *Effect of shifting the Centre of Pressure in a Double-lifting Aeroplane.*

In this paper it has been shown by considering the equations of motion that in the general case of a double-lifting aeroplane where

neither the front nor the rear planes are neutral and when the machine is moving uniformly the shifting of the centre of pressure will have the effect of either depressing or elevating the flight-path.

2. *The Steering of an Aeroplane in a horizontal circle.*

Dr. H. Reissner tackled the problem of lateral steering in 1910–11 and it was then tackled by Dr. G. H. Bryan. The present paper is a further investigation on the subject. By considering the equations of motion in the case of a symmetrical aeroplane such as commonly exists in practice, relations between the propellar thrust and the components of the air resistances have been established.

H. K. Sen, Allahabad.

Polytropic Gas Spheres with Variable Index

A sphere of perfect gas is considered, whose polytropic index n, defined by $\frac{d}{dr}(\log P) = \left(1 + \frac{1}{n}\right)\frac{d}{dr}(\log \rho)$, P and ρ being respectively the pressure and density at a distance r from the centre, varies in any manner from shell to shell between the limits n_1 and n_2. It is shown from quite general considerations that quite a number of properties of the variable polytrope, besides the gravitational potential energy, attain their extremal values in the limiting uniform polytrope of index n_1 or n_2. Using Candler's results in M. N. **100**, 14, 1939, fresh intermediate properties have been deduced for the variable polytrope, and alternative proofs have been given of Chandrasekhar's theorems on the central value of the ratio of the radiation to gas pressure. The polytropic index has been shown to increase monotonically from the centre of a variable polytrope to its surface and the signs of the variation of the other important physical variables have been found. Two new integral theorems have been proved on the central potential and temperature of a perfect gas sphere in gravitational equilibrium.

M. Zakiuddin, Muslim University, Aligarh.

A new method for evaluating the Vibration Constants of a Diatomic Molecule

The constants associated with a diatomic molecule can be calculated after the wave numbers of band heads, emitted by these molecules, have been fitted in a quantum vibrational scheme.

The equation is

$$\nu = \nu_e + [\omega_e'(v'+\tfrac{1}{2}) - x_e\omega_e'(v'+\tfrac{1}{2})^2] - [\omega_e''(v''+\tfrac{1}{2}) - x_e\omega_e''(v''+\tfrac{1}{2})^2]$$

and according to the old quantum theory we get for 0,0 the sequence of band heads,

$$\nu_{\text{head}} = \nu_0 + (a'-a'')\,v'' - (b'-b'')\,v''^2 + (c'-c'')v''^3$$

For 0, 1 sequence,

$$\nu_{\text{head}} = (\nu_0 - a' - b' - c') + (a' - a'' + 2b' + 3c')v'' - (b' - b'' + 3c')v''^2 + (c' - c'')v''^3.$$

where,

ν_{head} is wave number for 0,0 band head,

v'' is the final vibration quantum number,

a' and a'' are vibrational constants,

b' and b'' are anharmonic constants,

c' and c'' are correction terms for anharmonicity in the initial and final states.

The method of Birge and Shea is generally employed to get quadratic and cubic equations to represent the band heads mathematically. Recently Mr. Sulaiman Kerawala of the Aligarh Muslim University has devised a new method for fitting polynomials. This method can be employed to fit the wave numbers instead of the old method.

Bands of the calcium fluoride molecule have been fitted by me in mathematical scheme using both methods and old quantum mechanics and the superiority of the second method over the first is shown. New Quantum mechanical calculations can also be made.

M. V. Jambunathan, Maharaja's College, Mysore.

1. *Probability of the "Obtuse"*

In an article contributed to *The Mathematical Gazette*, (October 1939) Mr. C. O. Tuckey mentions, without proof, that the probability of drawing an obtuse-angled triangle at random varies from ·57 to ·75. In the present paper various values of the probability are deduced based on different hypotheses about "equally probable" cases.

2. *Mathematical Measure of a Favourable Set of Group Minima.*

In this paper an attempt is made to indicate mathematically in what manner the chances of securing a pass in an examination by a candidate who can score any percentage between 30 and 45 are enhanced by the introduction of a particular set of group minima.

A. A. Krishnaswami Ayyangar, Maharaja's College, Mysore.

1. *An improved formula for the Coefficient of Contingency.*

In statistics, the coefficient which summarises the mutual dependence of two attributes of the elements of a universe is called a coefficient of contingency. Two such coefficients C and T due respectively to Pearson and Tschuprow are well known. T is usually considered as an improvement on C. But we remark that T does *not remedy the defect* in C *in a* $s \times t$—*fold classification* when $s \neq t$. We propose a better coefficient K, which is related to C and T by the equations:

$$K^2 = \frac{c^2}{(1-c^2)(t-1)} = \frac{T^2\sqrt{(s-1)}}{\sqrt{(t-1)}}, \; (s \geqslant t)$$

K has the advantage of a unique upper limit, unity, which can be actually reached without making the classification too fine and thus introducing errors of sampling.

It is shown that the maximum value of T is $\sqrt[4]{\left(\frac{t-1}{s-1}\right)}$ and so, by a proper adjustment of t with respect to s, the value of the coefficient can be made less than any assigned small value. T will thus give spurious results.

2. *Fiducial Probability.*

There are two theories in Statistical Estimation, one named Fiducial Probability by R. A. Fisher, who is peculiarly fond of using ordinary words in extraordinary senses, and the other started by E. B. Wilson under the notion of Confidence Intervals. This paper attempts at a clarification of the relative merits of the ideas underlying these two almost synonymous names.

3. *Incompatibility of k-statistics.*

Tschuprow called G the presumptive value of a statistic F when the mathematical expectation of G is equal to F. In 1927, Mr. Bertelson worked out, according to Tschuprow's definition, the presumptive values of the semi-invariants, and these were introduced

later by Fisher as his k-statistics. By a very complicated method, Mr. Bertelson has proved the incompatibility of k_4, i.e., it does not correspond to any real set of observations. We now give a simple proof of this incompatibility, when for a sample of size n, β_2 is less than $(n^2+4n-9)/(n^2-1)$.

4. *Conventions in Mathematics.*

There are two types of conventions, (1) linguistic, and (2) logical. Assigning numbers to persons and things belongs to the former type, while the latter is implied in 'plus' and 'minus'. A convention does not possess the 'naivete' of an axiom that disarms suspicion. Custom is a blind unthinking adherence to something, which may be the result of a convention, whereas a convention is the result of a conscious and deliberate preference. The conventions 'plus' and 'minus' reflect most appropriately an essential dualism in nature. In mathematics, they represent dimensional directions.

Conventions are not mere peace-makers. They are fruitful stimuli for the advancement of thought. They led Mobiüs to the discovery of solids without volumes.

Very few elementary mathematical text-books formulate clearly and explicitly the most useful conventions, nor are they consistent in the adoption of conventions. At the threshhold of analytical geometry, we find the positive side of a line determined in at least three different ways. As regards the standard form of the equation of a line, Salmon and Askwith make the co-efficient of y positive, and Casey makes the absolute term positive, while recently Mr. J. M. Child makes the coefficient of x positive. It is desirable that we standardise one of these conventions, at least in the interests of students taking public examinations.

Another useful convention which is worth universal adoption is to regard a triangle not in the finite and restricted manner of old Euclid but in the more comprehensive way of trigonometry, in which the sides and angles are determined only by the well known sine-rule and angle-sum properties. The advantages of this neglected principle were pointed out by H. W. Richmond and H. Lob a dozen years ago but nobody seems to have taken the matter seriously.

5. *Inequalities and Unemployment.*

The paper starts with the paradoxical notions of inequality which are prevalent among economists. It introduces the concept of

'symmetric groups' of income-receivers and proposes a mathematical criterion to distinguish between over-employment and under-employment based on an empirical quadratic law, $k(x-a)(x-b)$, which seems to be eminently fitted for the consideration of the inequalities in question. Mathematics apart, it is suggested that a reform based on psychological and ethical lines will sooner solve the problem of inequalities than any purely economic measures.

M. R. Doresamiengar, Maharaja's College, Mysore

Parameters of 'Personal Distribution'

The study of parameters is important in any branch of Economics and particularly in 'personal distribution.' Thus levels of incomes are determined not only by the number of able-bodied men but also by the degree of industrialisation, volume of trade, facilities for migration from employment to employment and other factors, assessable and non-assessable. The analytical laws of incomes *in vogue* may be taken as the spade work in the field and it is proposed to work out the feasible parametric forms on the lines of Moore's 'Synthetic Economics.' Dr. H. Hotelling's work on parameters in Statistics is the chief source of inspiration of this paper.

Hansraj Gupta, Government College, Hoshiarpur

The Calendar

In this paper the principles on which the Christian, Vikrama and Hijri calendars are based are explained and the nature of the corrections required in each case discussed.

R. Krishnamurti, Hyderabad

Some facts about the Saros and the Metonic cycle.

In this paper it is discussed whether the Saros and the metonic cycle could have been the results of calculation instead of a purely observational result as it is commonly supposed.

K. R. Gunjikar, Bombay.

Some observations on the February comet.

SYMPOSIUM ON GROUP THEORY

Report on the Combinatorial Theory of Groups

BY

Prof. F. W. LEVI, (*Calcutta*)

Opening the general discussion on theory of groups, Prof. Levi pointed out that the automorphisms of a system of any kind form a group, and that every group can be considered as a group of automorphisms. This inter-connection seems to be the reason for the central position of the theory of groups. Starting from the idea of automorphism,—*i. e.* of generalised permutations—a review was given of the development of the theory in recent times.

Besides this main-line of progress, a certain by-way has got enhanced importance, where the idea of *combination* prevails. This theory is the subject of the report. The elements of a group are constructed by means of *generators* $a, b, c, \ldots$ which may be finite or infinite in number. With the generators and their *inverse* symbols a^{-1}, b^{-1}, c^{-1}, ... one forms *words* by arbitrary combination, e.g. ab, bb, $a\ b\ a^{-1}\ b^{-1}$, ...; these words represent group-elements, and their multiplication is effected by putting together the corresponding words. Such words like aa^{-1}, $a^{-1}a$, bb^{-1}, ... are called *trivial relations*; they stand for the unit element 1. By striking out and inserting trivial relations, the group-element is not altered; using these operations, one can transform every word either into 1, or into a word which is free from trivial relations. These words and the symbol 1 are called reduced words. One can prove that they form a group which is called the *free group* F generated by a, b, c, ... Let N be a normal subgroup of F and G = F/N ; consider the class a of elements of F which by the homomorphism F→G are mapped on the same element of G. Starting from any element of a, one gets all the other ones by repeated inserting and striking out of words which represent elements of N (in particular trivial relations; since they represent 1); these words are called the *relations* in G. One can obtain all of them from a certain subset, the *generating relations*, by successive composition and transformation. Every group is homomorphic to a free group; thus it can be constructed by generators and generating relations. It is therefore uniquely determined by a free group F and a normal subgroup in it. This representation is

not unique, since e. g. F generated by $a, b, c, \ldots$ is also generated by $a, ba, c, \ldots$ but the number (finite or transfinite) of the free generators of F is an invariant. Let F, generated by n free generators, admit a representation by m generators with q generating relations, and let q be the minimal number for those generators, then $m = n + q$. If F′ is a subgroup of F, then F′ is a free group. The numbers n and N of the free generators of F and F′ and the index j of F over F′ are interconnected by $j + N = 1 + n\,j$. This formula determines N when j is finite but not when j is infinite. If j is infinite, and F′ a normal subgroup, $N = nj$ holds. From this formula is easily seen that free groups with 2 (or more) generators have subgroups with an enumerable (and therefore also with any finite) number of generators. The words where every generator occurs as often as its inverse symbol, represent the elements of the commutatorgroup $C_1(F) = (F, F)$ of F. There exists an infinite sequence $C_2(F) = (C_1, F)$; $C_3(F) = (C_2, F)$ ... of different commutatorgroups (unless F has a single generator) which have 1 as the only common element. The homomorphism $F \to G$, maps $C_m(F) \to C_m(G)$, but the commutatorgroups of G may not all be different. In this way the theory of free groups is applied to other groups. The methods used in the combinatorial theory of groups can be classified into (i) combinatorial, (ii) group theoretical, (iii) topological methods.

1. Operating with combinatorial methods, one considers what occurs when a number of words are combined and relations are inserted and struck out; the simplifications which can be effected by a change of the system of generators are important. These investigations are often tricky, but it seems that they cannot be completely avoided. The most important auxiliary notion is the "length" (i. e. the total number of symbols) of a word, the length is mostly used as a perameter for mathematical induction.

2. Group theoretical methods can be used as the theory advances and group theoretical results are obtained. The most fundamental operation is the homomorphism in particular the homomorphism "making G abelian", i.e. mapping G on the factorgroup of the commutatorgroup.

3. The topological methods are combinatorial methods where the combinations on a linearly ordered set are replaced by combinations on a manifold of more dimensions. This way of procedure tallies better with the nature of the theory since the "relations" are invariant for cyclic permutations. The groups represented by

generators and generating relations admit a topological representation by "group-images" of various kinds, and these can be treated by methods of the combinatorial topology. The report dealt chiefly with the group-images and with the theory of the free products in which all the different methods of combinatorial theory of groups are used. As an important application, the theory of knots and chains was mentioned.

Theory of p-groups

BY

M. A. JABBAR, M. Sc., (*Calcutta*).

Every Abelian group of finite order is the direct product of characteristic subgroups which are p-groups (i. e., groups of prime power orders). Thus the study of Abelian groups reduces to that of Abelian p-groups. These have a basis representation which is unique (in the sense of isomorphism). The structure of non-Abelian groups is more complicated, but there exists some important similarities with the Abelian case, which offer an opportunity for starting a separate investigation for them. Burnside has shown [1] that every p-group has a minimal basis, which is not unique. In his paper, Hall has shown [2] that there is a class of p-groups (termed regular), which admit unique basis representation similar to Abelian p-groups. For every exponent n there exists only a finite number of p-groups of order p^n (p running over all the prime numbers) which are not regular, whereas the number of regular p-groups is infinite.

If H and K are any two groups, then (H, K) is the group generated by elements of the form $(h, k) = h^{-1} k^{-1} hk$, where h is an element of H and k is an element of K. The series $G = H_1 > H_2 > \cdots\cdots > H_c > H_{c-1} = 1$, where $H_i = (H_i - 1, G)$, exists for any p-group G (more generally for any nilpotent group) [3] and is known as the lower central series. The length c of the series is called the class of the group.

An element P of a group G is said to be a commutator of weight 1 and $(P, Q) = P^{-1} Q^{-1} PQ$ is a commutator of weight 2; generally if S is a commutator of weight w_1 and T is another commutator of weight w_2, then the complex commutator (S, T) is

[1] *Proc. Lond. Math. Soc.* 1913.
[2] *Proc. Lond. Math. Soc.* 1934.
[3] *H. Zassenkaus*: *Lehrbuch der Gruppentheorie*, I. Bd. IV, §5.

said to be a commutator of weight w_1+w_2. The generators of H_i in the series given above are of weight i and the subgroup H_i is a complex commutator group of weight i. If $f(G)$ is any complex commutator subgroup of a p-group G and if it is of weight w, then $f(G) \leqslant H_w$. In particular if w is $>c$, the class of G, then $f(G)=1$.

If P and Q are any two elements of a group G, then for any exponent α and any pre-assigned integer m

$$(PQ)^{p^\alpha} = P^{p^\alpha} Q^{p^\alpha} R_3^{n_3} R_4^{n_4} \ldots\ldots\ldots R_r^{n_r} T_m.$$

where R_i's: are various complex commutators in P and Q arranged in order of non-decreasing weights, otherwise arbitrary;

T_m belongs to H_m;

and n_i's are positive integers depending on p^α and the weight w_i of R_i.

In the case of a p-group (more generally a nilpotent group) if the weight of R_{r+1} is greater than the class of the group generated by P and Q, then R_{r+1} and $R_{s>r+1}$ are all equal to 1. Hence in these cases

$$(PQ)^{p^\alpha} = P^{p^\alpha} Q^{p^\alpha} R_3^{n_3} \ldots\ldots\ldots R_r^{n_r}.$$

The groups for which $n_i = p^\alpha$, for $i=3, 4, \ldots\ldots r$, are said to be "regular" p-groups of this kind have the characteristic property that for every exponent α, $P^{p^\alpha} Q^{p^\alpha} = R^{p^\alpha}$, for a suitable element R of the group and that $P^{p^\alpha} = Q^{p^\alpha} = 1$ implies $(PQ)^{p^\alpha} = 1$. So the p_α th powers of the elements of regular group G form a characteristic subgroup $\mho_\alpha$ and that the elements which satisfy the relation $P^{p^\alpha} = 1$, form another characteristic subgroup Ω_α, for $\alpha = 0, 1, 2, \ldots \ldots \mu$, where p^μ is the highest order of the elements of G.

If p^{ω^α} is the order of the factor group $\Omega_\alpha/\Omega_{\alpha-1}$, then $\omega_1 \geqslant \omega_2 \geqslant \ldots \geqslant \omega^\mu$ holds; and if $\mu_\beta =$ the number of $\omega_\alpha \geqslant \beta$, then μ_i's are invariants for G and $p^{\mu_1}, p^{\mu_2}, \ldots\ldots p^{\mu_{\omega_1}}$ are the orders of the basis elements $P_1, P_2, \ldots\ldots P_r$ respectively. So the group may be represented by the invariants $(\mu_1, \mu_2, \ldots\ldots \mu_{\omega_1})$ which is known as the type of the group as in the case of Abelian groups of prime power orders. But though to every type $(\mu_1, \mu_2 \ldots\ldots \mu_\gamma)$ there exists only one Abelian group (one in the sense of isomorphism), it is not generally so in the case of regular p-groups.

Characters of the Symmetric Group

BY

DR. M. ZIAUD DIN, *Lahore*

The characters of the symmetric group, play an important part in higher algebra, statistics, quantum mechanics and nuclear physics. Investigations on this subject have been carried out chiefly by Frobenius, Littlewood and Richardson, Murnaghan and M. Ziaud Din.

The author has published the tables of group characters of the symmetric group of degrees 11, 12 and 13 in the *Proceedings London Math. Soc.* (2) 39 (1935) and 42 (1937), and the tables of degrees 1—10 have been published by other authors quoted above.

Frobenius was the first who took up the construction of tables of group characters using the following method. Let S be an operation of the symmetric group on n symbols, containing α_1 cycles on one symbol, α_2 cycles on two symbols and so on, such that $n=\alpha_1+2\alpha_2+3\alpha_3+\cdots$; the numbers $\alpha_1, \alpha_2, \alpha_3, \cdots$ define the class ρ to which S belongs and the coefficient of $\pm\, x_1^{\lambda_1} x_2^{\lambda_2} \cdots\cdots x_n^{\lambda_n}$, will give the character of S in an irreducible representation defined by the sequence $\lambda_1, \lambda_2, \lambda_3, \cdots\cdots \lambda_n$ in the product

$$(x_1+x_2+\cdots x_n)^{\alpha_1}(x_1^2+x_2^2+\cdots)^{\alpha_2}\cdots(x_1^n+x_2^n+\cdots)^{\lambda_n}\cdot A(x_1\, x_2\cdots x_n).$$

where $A(x_1\, x_2\cdots)$ denotes the simple alternant.

Frobenius method is very laborious. Simpler and quicker methods have been developed by Littlewood and Richardson, and Murnaghan, a detailed account of which will be found in D. E. Littlewood's Book 'The theory of group characters (1940)' and in Murnaghan's "The theory of group representation" (1938). The methods of Littlewood and Richardson, are based on Immanants and S-functions, which may be briefly described as follows.

Immanants of a Matrix

Let $[a_{st}]$ be a matrix of order n^2, S being any permutation $e_1, e_2, e_3 \cdots e_n$ of the numbers 1, 2, 3, $\cdots n$ and

$$P_S = a_{1e_1}\cdot a_{2e_2}\cdot a_{3e_3}\cdots\cdots a_{ne_n}.$$

There are $n!$ products P_S corresponding to the $n!$ permutations of the symmetric group.

If $\chi^{(\lambda)}$ denote the character of the symmetric of order $n!$, corresponding to the partition $(\lambda)=(\lambda_1, \lambda_2, \cdots \lambda_p)$ of n, the immanant of the matrix $[a_{st}]$ corresponding to the partition (λ) is defined by

$$|a_{st}|^{(\lambda)} = \sum_S \chi^{(\lambda)}(S) \cdot P_S,$$

the summation being with respect to the $n!$ permutations of the symmetric group.

Since the character $\chi^{(n)}$ is unity for every operation of the group, and the character $\chi^{(1^n)}$ is $+1$ for a positive permutation and -1, for a negative permutation, the permanent and determinant of a matrix are special cases of immanants. Thus

$$\overset{+}{|}\, a_{st} \,\overset{+}{|} = |a_{st}|^{(n)}$$

$$|a_{st}| = |a_{st}|^{(1^n)}.$$

S-Functions

These are called S-functions after Schur, who first dealt with symmetric functions in relation to groups. S-functions may be defined from the immanant of a matrix as follows:

If (λ) is a partition of r, with the parts in descending order, the S-function denoted by $\{\lambda\}$ is defined by the equation

$$r!\ \{\lambda\} = |z_r|^{(\lambda)}$$

where the matrix $[z_r]$ is

$$[z_r] = \begin{vmatrix} S_1, & 1, & 0 & 0 & \cdot & 0 \\ S_2, & S_1, & 2 & 0 & \cdot & 0 \\ S_3, & S_2, & S_1 & 3 & \cdot & 0 \\ S_{r-1} & \cdot & & \cdot & \cdot\ S_1, & r-1 \\ S_r, & \cdot & & \cdot & \cdot & S_1 \end{vmatrix}; \quad S_r = \sum x_1^r. \quad \text{(power-sum, symmetric function).}$$

For a symmetric group of order $r!$, if ρ denotes the class of order h_ρ, and $\chi_\rho^{(\lambda)}$ the character for the class ρ, then S-functions satisfy the relations.

$$(1) \quad r!\ \{\lambda\} = \sum h_\rho\, \chi_\rho^{(\lambda)}\, S_\rho$$

$$(2) \quad S_\rho = \sum \chi_\rho^{(\lambda)} \{\lambda\}.$$

where $S_\rho \equiv S_1^{\alpha_1} S_2^{\alpha_2} S_3^{\alpha_3} \cdot \cdot \cdot \cdot$

S-functions can also be expressed as quotients of two alternants, *i.e.* general alternant ÷ simple alternant. In this form these functions have been already dealt with by several writers, as has been described by the author in the paper on 'Determinantal Symmetric Functions' *Proc. Edinburgh Math. Soc.* (2) 4 Part I 47.

Tables of the characters of the symmetric group can thus be constructed using the properties of S-functions given in Littlewood's book. The characters can be checked by using the orthogonal properties

$$\sum \chi_\rho^{(\lambda)^2} = h/h_\rho$$

$$\sum \chi_\rho^{(\lambda)} \chi_{\rho'}^{(\lambda)} = 0$$

where h is the order of the group.

S-functions and group characters are also applicable to Invariant matrices as I have shown in my paper 'Invariant matrices and S-functions' *Proc. Edin. Math. Soc.* (2) 5 Part I 43.

Symposium On Fourier Integrals and Transforms

BY

DR. R. S. VARMA, *Lucknow University.*

1. *Introduction.* The theory of Fourier Integrals is closely allied to the theory of Fourier series. A function $f(x)$, of period $2\pi\lambda$, is represented by the Fourier series

$$f(x)=\tfrac{1}{2}\,a_0+\sum_{n=1}^{\infty}\left(a_n\cos\frac{nx}{\lambda}+b_n\sin\frac{nx}{\lambda}\right), \quad \dots \quad (1\cdot1)$$

the coefficients a_n and b_n being given by

$$a_n=\frac{1}{\pi\lambda}\int_{-\pi\lambda}^{\pi\lambda}f(t)\cos\frac{nt}{\lambda}\,dt, \quad \dots \quad (1\cdot2)$$

$$b_n=\frac{1}{\pi\lambda}\int_{-\pi\lambda}^{\pi\lambda}f(t)\sin\frac{nt}{\lambda}\,dt. \quad \dots \quad (1\cdot3)$$

With the aid of (1·2) and (1·3), (1·1) may be written as

$$f(x)=\frac{1}{2\pi\lambda}\int_{-\pi\lambda}^{\pi\lambda}f(t)\,dt+\sum_{n=1}^{\infty}\frac{1}{\pi\lambda}\int_{-\pi\lambda}^{\pi\lambda}f(t)\cos\frac{n\,(x-t)}{\lambda}\,dt \quad \dots \quad (1\cdot4)$$

If we put $\frac{n}{\lambda}=u$, $\frac{1}{\lambda}=\delta u$, and make λ tend to ∞, the sum on the right of (1·4) passes formally into the form

$$f(x)=\frac{1}{\pi}\int_0^{\infty}du\int_{-\infty}^{\infty}f(t)\cos u\,(x-t)\,dt \quad \dots \quad (1\cdot5)$$

This is *Fourier's Integral Formula.*

If $f(x)$ is an even function, Fourier's integral becomes

$$f(x)=\frac{2}{\pi}\int_0^{\infty}\cos xu\,du\int_0^{\infty}\cos ut\,f(t)\,dt, \quad \dots \quad (1\cdot6)$$

the term involving $\sin ut$ vanishing identically. This is *Fourier's cosine formula.* If now we write

$$F_c(u)=\sqrt{\left(\frac{2}{\pi}\right)}\int_0^{\infty}\cos ut\,f(t)\,dt, \quad \dots \quad (1\cdot7)$$

then (1·6) gives

$$f(x)=\sqrt{\left(\frac{2}{\pi}\right)}\int_0^{\infty}\cos xu\,F_c(u)\,du, \quad \dots \quad (1\cdot8)$$

There is therefore a reciprocal relation between the functions $f(x)$ and $F_c(x)$. A pair of functions connected by formulae (1·7) and (1·8) are known as *Fourier cosine transforms* of each other.

Similarly from (1·5), if $f(x)$ be odd, we obtain

$$f(x) = \frac{2}{\pi} \int_0^\infty \sin xu \, du \int_0^\infty \sin ut \, f(t) \, dt, \quad \dots \quad (1\cdot 9)$$

and from this the reciprocal formulae

$$F_s(u) = \sqrt{\left(\frac{2}{\pi}\right)} \int_0^\infty \sin ut \, f(t) \, dt \quad \dots \quad (1\cdot 10)$$

and

$$f(x) = \sqrt{\left(\frac{2}{\pi}\right)} \int_0^\infty \sin xu \, F_s(u) \, du \quad \dots \quad (1\cdot 11)$$

The relation (1·9) is known as the *Fourier's sine formula* and a pair of functions satisfying (1·10) and (1·11) are called *Fourier sine transforms* of each other.

The *exponential* form of Fourier's formula, given by Cauchy, is

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^\infty e^{-ixu} \, du \int_{-\infty}^\infty f(t) \, e^{iut} \, dt, \quad \dots \quad (1\cdot 12)$$

which leads us to the unsymmetrical formulae

$$F(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty f(t) \, e^{iut} \, dt, \quad \dots \quad (1\cdot 13)$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty F(u) \, e^{-ixu} \, du \quad \dots \quad (1\cdot 14)$$

If $f(x)$ is even, $F(x) \equiv F_c(x)$ and if $f(x)$ is odd, $F(x) \equiv i \, F_s(x)$.

It is interesting to see that the pair of formulae, known as *Mellin's inversion formulae*,

$$F(s) = \int_0^\infty x^{s-1} f(x) \, dx,$$

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s) \, x^{-s} \, ds$$

leads us to the fact that $\sqrt{(2\pi)} \, e^{cx} f(e^x)$ and $F(c+ix)$ are Fourier transforms of each other in the sense of (1·13) and (1·14).

2. The subject of Fourier Integral is important both from the point of view of pure and applied mathematics. The so-called Laplace transform and its complex analogue now called the Fourier

transform were both known to Laplace as well as to Fourier, who used them in connection with the problem of the conduction of heat in the half plane.

Fourier series may be used in the analysis of a complex wave form such as would occur on a gramophone record. The coefficients a_n and b_n would then be proportional to the amplitudes of the various harmonics occurring as over tones in the original sound. The Fourier integral would then be the sum total of the contributions due to an irregular wave - form of all adjacent frequencies over the entire frequency spectrum. It is implied that the component frequencies exist throughout time, past and present, and, at a particular epoch, their phases are such that they describe the required wave - form.

3. In the theory of Fourier series Riemann Integration is sufficient to give all the theorems when we proceed from a given function to its Fourier coefficients, but when we go to the function from a set of coefficients, Riemann integration does not help us and we have to take into consideration integration in a wider sense, viz. that of Lebesgue. Further the Fourier coefficients are a discrete non-periodic set of numbers while the function is defined for a continuous infinity of values but is periodic and hence need only be given within a single period. In the theory of the Fourier Integral, it is not possible to segregate the difficulties into two such categories. If the function to be expanded is $F(u)$ of (1·13), then $f(u)$ plays the part of the set of Fourier coefficients. The formal similarity between (1·13) and (1·14) shows that it is not possible to say which of f and F is the function to be expanded and which is the set of coefficients. Both are non-periodic coefficients defined over continuous infinite ranges. In spite of this, the difficulties are there in the theory of Fourier Integrals and we have to construct its theory on the basis of Lebesgue integration.

4. *Functions of class* L. A direct consideration gives that if $f(t)$ is integrable in the Lebesgue sense over $(-\infty, \infty)$ and is of bounded variation in an interval including the point x, then

$$\tfrac{1}{2}\{f(x+0)+f(x-0)\}=\lim_{u\to\infty}\frac{1}{\pi}\int_0^u du\int_{-\infty}^{\infty} f(t)\cos u(x-t)\,dt; \quad \ldots(4\cdot1)$$

as a particular case, if $f(t)$ is continuous,

$$f(x)=\frac{1}{\pi}\int_0^{\infty} du\int_{-\infty}^{\infty} f(t)\cos u(x-t)\,dt. \quad \ldots\ (4\cdot2)$$

An interesting result for the class of functions under consideration is that, if $f(t)$ is $L(-\infty, \infty)$, then the integral on the right of

(4·2) is summable (C, 1) to $\frac{1}{2}\{f(x+0)+f(x-0)\}$ wherever this expression has a meaning.

5. *Functions of class* L^2. We have seen that though the reciprocal formulae giving $f(x)$ and $F_c(x)$ or $F_s(x)$ (the cosine or sine transforms) are symmetrical, yet the conditions which these functions satisfy are quite different. Plancherel, using the theory of mean convergence, has developed the transform theory in which he uses functions of the class L^2. Thus, if $f(x)$ belongs to $L^2(0, \infty)$, then the formulae for the cosine transform hold in the sense that, as $a \rightarrow \infty$, the integral

$$g_c(x) = \sqrt{\left(\frac{2}{\pi}\right)} \int_0^a \cos xt \;\; f(t)dt.$$

converges in the mean over $(0, \infty)$ to a function $g(x)$ of the class

L^2 $(0, \infty)$ and $$f_c(x) = \sqrt{\frac{2}{\pi}} \int_0^a \cos xt \;\; g(t)\, dt.$$

converges in mean to $f(x)$.

We have almost everywhere

$$g(x) = \sqrt{\left(\frac{2}{\pi}\right)} \frac{d}{dx} \int_0^\infty f(t) \frac{\sin xt}{t} dt,$$

and,

$$f(x) = \sqrt{\left(\frac{2}{\pi}\right)} \frac{d}{dx} \int_0^\infty g(t) \frac{\sin xt}{t} dt.$$

The corresponding theorem for sine transforms holds with $\cos xt$ replaced by $\sin xt$ and $\sin xt$ by $1 - \cos xt$.

6. *Parseval formulae.* Suppose now that $g_1(x)$ and $g_2(x)$ are Fourier cosine transforms of $f_1(x)$ and $f_2(x)$, we have, both for transforms of L and of L^2,

$$\int_0^\infty g_1(x)\, g_2(x) \;\; dx = \int_0^\infty f_1(x) \;\; f_2(x) \;\; dx$$

and

$$\int_0^\infty \{g_r(x)\}^2 dx = \int_0^\infty \{f_r(x)\}^2\, dx \qquad r = 1, 2.$$

The formulae remain valid even when $g_1(x)$ and $g_2(x)$ are Fourier sine transforms of $f_1(x)$ and $f_2(x)$ respectively.

These are analogous to Parseval's formulae in the theory of Fourier series

$$\frac{1}{\pi} \int_0^{2\pi} \{f(x)\}^2 dx = \tfrac{1}{2} a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

7. *Self-reciprocal Functions.* The time at my disposal does not allow me to deal with the various forms of general Tauberian theorems, or the Lambert-Tauber theorem and the de la Valee Poussin-Hadamard theorem concerning the distribution of primes, or the theorem that a function which has a spectrum has a positive spectrum, or with the recent and very interesting and fruitful investigation of Titchmarsh on expansions in Eigenfunctions (*Q. J. Math.* vols. 11, 12 and *J. L. M. S.* vol. 14). But I would like to take some time more in giving a passing reference to the theory of self-reciprocal functions as this subject has been engaging my attention for the last few years. In the general sense, a function $f(x)$ is said to be *self-reciprocal in the Hankel Transform* or is said to be R_ν (using an abbreviation given by Hardy and Titchmarsh) when

$$f(x)=\int_0^\infty f(y)\sqrt{(xy)}\,J_\nu(xy)\,dy. \qquad \ldots \quad (7\cdot1)$$

For $\nu=-\frac{1}{2}$, this reduces to

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(y)\cos xy\,dy,$$

and for $\nu=\frac{1}{2}$,

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(y)\sin xy\,dy.$$

In the last two cases, $f(x)$ is said to be *its own cosine* or *sine Transform.*

A formal solution of (7·1) is given by

$$f(x)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}2^{\frac{1}{2}s}\,\Gamma\left(\tfrac{1}{2}\nu+\tfrac{1}{2}s+\tfrac{1}{4}\right)\psi(s)\,2^{-s}\,ds,$$

where $\psi(s)=\psi(1-s)$.

The remarkable memoir of Hardy and Titchmarsh (*Quarterly Journal of Mathematics* (Oxford series), Vol. 1 (1930), pp. 196-231) is very useful in this direction.

During the last ten years, a number of formulae connecting different classes of self-reciprocal functions have been discovered. In India, the works of Dr. B. Mohan stand conspicuous in this direction. Recently Mr. P. C. Mittal, a research scholar at the University of Lucknow, has undertaken the task of correlating a self-reciprocal function with Laplace's integral. He has so far succeeded in giving two theorems and his investigations will appear in the *Journal of the Indian Mathematical Society.*

SYMPOSIUM ON THE ORIGIN OF THE SOLAR SYSTEM

Chairman: PROF. A. C. BANERJI, (*Allahabad*)

D. N. MOGHE, *R. R. College*, *Bombay*, opened the discussion.

The first attempt to give a rational theory of the origin of the solar system was by Kant. He thought that the planets were born out of a primeval nebula. But his was a purely non-mathematical theory. Laplace built up Kant's hypothesis on mathematical lines. It was supposed that the primeval nebula shed a ring of matter every time it contracted due to rotational motion. Every such ring condensed into a planet. It has been shown by Jeffreys that such a ring instead of condensing into a single mass would break up into small bodies like the asteroids. Moreover, the angular momentum possessed by the Sun is 1/60 of the total angular momentum of the whole system. Laplace's theory fails to explain this fact and hence it is untenable.

Chamberlin and Moulton assumed that a passing star had a grazing encounter with the primitive Sun which released eruptive forces within it and shed a great amount of matter which formed the planets. The hypothesis put forth by Jeans was that the tidal attraction of the star was responsible for the ejection of matter, while Jeffreys supposed that there was a direct collision between the star and the Sun creating a filament which condensed into planets. The star went away along its hyperbolic path and the planets (or, the filament) was altogether under the gravitational influence of the Sun. But, in such a case, calculation showed that the angular momentum had such a trifling value that the size of the orbits could not be explained.

An advance over this was made by Lyttleton who supposed that the Sun was primarily a double star and the collision due to the near approach of an intruding star gave rise to a filament which ultimately broke up into different planets moving along different orbits. The different sizes of these orbits can be accounted for on this theory. But it has been shown by Luyten and Hoyle and Bhatnagar that the conditions for the escape of the companion and the intruder are difficult to fulfil.

In his presidential address to the Mathematics Section of the Indian Science Congress held at Madras in 1940, Prof. A. C. Banerji suggested that the Sun itself was the intruding star which had a close or grazing collision with the companion of a double star and that there was a planetary filament created between this companion and the Sun. This is, however, found to be untenable as the result of the recent development in this direction due to Dr. Spitzer and Prof. H. N. Russell.

A cursory review of all these different theories will lead us to the conclusion that the main objections against all these theories are purely of a dynamical nature. The outstanding difficulties are: (i) What is the proper source from which we can get the requisite angular momentum for the Sun and the planets? (ii) How can we account for the different sizes of the planets and their orbits? Unless a satisfactory hypothesis is put forward so as to explain these points no theory will find a firm ground.

But, quite recently, objections have been raised against the "filament theories", objections which are not dynamical but which fall in the domain of Physics. It must be a fact that matter composing the filament is at extremely high temperature so that gravitational attraction is too feeble to affect the enormous speed of the individual atoms. They would, therefore, escape to great distances at any rate beyond the attractive influence of the primitive Sun. On the contrary the process of radiation would cool the gaseous mass forming the filament effecting condensations in the filament. Laws of gas opacity can only decide which of the two will have the upper hand. Dr. Spitzer concludes that expansion being much faster than cooling, the filament is reduced to a mere envelope round the Sun which is absolutely incapable of forming the planets.

Quite recently, Lyttleton has proposed a theory on the principle of rotational instability to remove the necessary material from the star to form the planets. This is a revival of the fission theory in which the Sun is again a double star. The Sun breaks up into two bodies thus forming a triple system. He finds that such systems are frequent in nature.

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We have thus seen that the encounter or the collision theories do not satisfactorily explain the formation of the Solar System. Now, two alternatives are before us: whether to propose an altogether new theory, or, whether to revive an old one with the necessary improvements? Our president, Prof. Banerji is shortly going to enlighten us with his new theory. I for myself am of the opinion that a revival of the old nebular theory with the necessary improvements will lead us to a fruitful alley. It is well known that condensations are formed at different stages along the spiral arms of a nebula. These condensations are at different distances from the central body which is the primitive Sun. The process of successive cooling and contracting will reduce the enormous size of this primitive system to the present size of the solar system. As the process of cooling is a very slow one the age of the solar system will have a very high value which will perhaps agree with the geological estimate. It will be possible on this theory to explain the different sizes of the planetary orbits. Work on these lines is still in progress and when all the necessary results are obtained a paper on this subject will be communicated for publication.

C. P. S. MENON, (*Dehra Dun*)

who followed dealt briefly with the fundamental problem of the origin of the solar system and (1) the theories of rotational instability (2) tidal

theory of Jeans (3) the collision theory, and the objections to each of these. He remarked that the theory suggested by Prof. Moghe seemed to be open to the same objection as the old theory, viz. (*a*) how the critical lentecular shape could have fallen back to the spherical shape and (*b*) how the filaments shed through the equilibrium points could condense into detached masses instead of undergoing disruption due to gravitational instability. Prof. Banerjee's theory appeared to be a useful contribution provided that the amplitudes of pulsating cepheid variables could, according to actual data, be so big that the second order terms made the series divergent.

PROF. A. C. BANERJI. (*Allahabad*)

From time to time various theories of the Origin of the Solar System have been proposed, but none of them have stood the test. The planetesimal theory of Chamberlin and Moulton and the tidal theories of Jeans and Jeffreys in which the Sun and the intruding star narrowly missed each other were discounted by Russell on the ground that the weighted average of the angular momentum per unit mass for all the planets is found to be nearly ten times greater than the amount which the Star can possess due to its orbital motion relative to the Sun. Lyttleton examined mathematically Russell's suggestion that the Sun might have been a component of a binary star whose companion had been removed by a close encounter with a passing star. Luyten and Hill criticised Lyttleton's theory on the ground that the kinetic energy required to form the planets is very large and that only 6% of the planetary ribbon could be captured by the Sun. Luyten further pointed out that, if the intruding star was sufficiently massive to give the requisite kinetic energy, the Sun was unlikely to capture any part of the ribbon without itself being captured by the intruder. Very recently Bhatnagar has shown mathematically that at the middle of the encounter the Sun would be so near the intruding star that a collision or a close encounter between the two could hardly be avoided. From astrophysical considerations Spitzer proved that even if a planetary ribbon be formed by a close encounter or a grazing collision between two stars, it will be diffused in space without giving birth to the planets.

Ross Gunn has considered the case of a rapidly rotating star breaking up by fission possibly in the vicinity of another star. The main difficulty in this theory is that such close encounters in which one of the approaching stars is rotating with angular velocity almost equal to its critical value of fission are highly improbable. Very recently Lyttleton has also considered the question of planetary formation resulting from the break-up of a single stellar mass due to rotational instability. His theory is open to the same objection as his earlier theory of planetary formation depending on encounters.

It thus appears that no existing tidal theory can explain the origin of the Solar System. To obviate the difficulties of the tidal theories, the Sun in the present theory is supposed to be originally a part of a Cepheid variable of about nine times the Sun's mass, which oscillated with small amplitude. The

nearby passage of a star of about the Cepheid's mass increased the amplitude of the oscillations rendering it unstable. That this is possible has been mathematically shewn in this paper. Matter was consequently thrown out which condensed into the Sun and the planets. Sufficient energy for this ejection was available in the parent Cepheid. The Sun is shewn to have taken about two-fifths of the energy of the parent Cepheid. It has also been shewn that the encounter need not have been very close nor the intruding star have a very high velocity to give the requisite angular momentum to the Sun and its planets to escape from the parent Cepheid. It is to be noted that the two main difficulties of the theory, viz. of energy and angular momentum, have been met here.

According to this theory, the parent Cepheid would also have a planetary system round it, and the planetary systems in our universe would be much more frequent.

BUSINESS MEETING OF THE INDIAN MATHEMATICAL SOCIETY

1. The Society paid tributes to the memory of the late Mr. M. T. Naraniengar, and Dr. Sir Shah Mohammad Sulaiman, all the members standing.

2. The Reports of the Secretary, the Librarian and the Joint Editor were read and adopted.

It was agreed that a list of books and periodicals available in the Society's library at Poona should be printed as early as possible and made available to members.

3. Suggestions regarding the advisability of having a uniform rate of the present subscription of Rs. 10/- for all members were considered. It was resolved that there should be no change in our rates of subscription, and that life composition fees be not accepted during the period of the present war.

It was also resolved that if the subscription for each year was not paid by the 1st of March, they may be recovered by the issue of our publications or of a card by Value Payable Post.

4. It was also resolved that every application for admission as a member should be accompanied by a remittance of a year's subscription before consideration by the Committee.

5. A resolution was passed thanking the Bombay University for their generous gift of Rs. 200/- annually for 5 years and for the payment of arrears since the lapsing of the previous grant.

6. The offer of Dr. A. Narasinga Rao to institute a prize for the encouragement of research to be awarded at the biennial Conferences was accepted with thanks and the Managing Committee was asked to frame the necessary rules.

7. At the suggestion of Prof. Levi, the desirability of students who had completed a course of advanced study in one University being given facilities for work in other Universities was discussed.

It was agreed that

(i) Universities be requested to arrange for research scholars to be sent to specialists and that each University should pay the expenses of its students.

(ii) that Universities and Colleges be requested, whenever possible, to arrange special courses of lectures open to the post-graduate students of all Universities.

(iii) that intimation of such lectures be sent to Heads of departments of mathematics of various Universities, at least six months before the commencement of the lectures.

GLEANINGS

ROB ROY'S PROBLEM

A column of infantry one mile long is marching forward at a constant rate of speed. An officer, mounted on horseback, starts at the rear of the column and rides forward at a uniform rate of speed which is faster than that of the infantry. He rides until he catches the head of the column and then instantly turns back and rides again to the rear of the column, reaching the rear at precisely the moment it passes the point at which the head started when the officer began his ride. How far did the officer ride? No time is allowed for his turning around to ride back.

[Ans. $1+\sqrt{2}$ miles]

"......most so-called philosophers, when they venture into mathematics, sell us only *aegri somnia* (sick man's dreams) for philosophy.

CARL FRIEDRICH GAUSS.

DISCUSSION ON THE TEACHING OF MATHEMATICS IN SCHOOLS AND COLLEGES

Chairman: SIR ZIAUDDIN AHMAD.

Opening the discussion SIR ZIA-UD-DIN AHMAD (Aligarh) said that Mathematics was expanding and that in order to include the new developments it became necessary to take over more and more into the pre-university course. Thus Arithmetic could be finished before the high school stage, while Algebra up to the Binomial Theorem and the elements of solid Geometry and Trigonometry might be included in the school course. The idea of a function and the elements of the calculus should find a place in the Intermediate stage, where there must be a combination of the intuitive as well as the analytical methods of approach. In the B. A. course one had to keep in mind also the requirements of students who would specialize later in Physics, by including a study of differential equations, especially those of mathematical Physics. Of advanced studies in the M. A. classes, he said that students of one college might go over to other colleges for such lectures. All training colleges should arrange for courses and lectures for the teaching of particular subjects, and all teachers invited to attend them.

C. P. S. MENON (Dehra Dun) stressed the claims of Mechanics for a place in the various syllabuses. He was against the mixing up of the several subjects in a composite course.

S. K. ABHAYANKAR (Gwalior) said that it was necessary to follow the historic method. New knowledge must be correlated with the old by pointing out analogies between the two.

M. ZIA-UD-DIN (Lahore) pleaded for the inclusion of laboratory work in statistics for the Inter and B.A. students.

KAZI KHURSHEED AHMAD (Allahabad) said there was too much of mechanical copying by students and that the standard of students was going down. Discussions should be encouraged and something done to improve the methods of teaching.

D. D. KOSAMBI (Poona): We had to ask ourselves in each case for what purpose we were teaching and whom we were teaching. The language difficulty was partly responsible for our troubles. As an instance of muddled notions regarding Newton's third law, he instanced the explanation of some pupils that though a horse and cart pulled each other with an equal force, the cart moved forward because the horse was the stronger of the two !

G. R. GUNJIKAR (Bombay) said that the setting of question papers and the marking of scripts should receive greater consideration than had been given to them. He was for an early introduction of algebraic symbolism.

B. RAMAMURTI (Ajmer) said that the utility aspect of mathematics should not override the artistic, and that we owed a duty in the matter of educating our teachers, as important as that we owed to students.

RAM BEHARI (Delhi): The historical background should be used to make the subject more interesting. Few students knew that Algebra was derived from Alkhowarezmi, or that the credit of solving the Pellian equation should go to Bhaskara. Fully rigorous proofs could not be given at the B. A. stage, but we should tell the student the conditions under which the result fails.

A. C. BANERJI (Allahabad) complained that Mechanics was taught in a haphazard manner. Physical principles were not clearly understood and much time was spent in a mechanical solution of problems.

G. R. SETH (Delhi) wanted Statistics and Actuarial Science, in the collegiate course so as to fit the student for employment after taking his degree.

RAM DHAR MISRA (Lucknow) complained that teaching was often limited to probable questions. He wanted that teaching should keep pace with the developments in the subject.

A. NARASINGA RAO (Annamalainagar) suggested that the columns of *The Mathematics Student* might be used as a pillory for exposing mistakes in university examination papers and in books, so as to ensure a high standard in both. He regretted that in bringing out text books, experienced teachers in India were not coming forth as freely as younger men in the profession. While theoretical mathematics received fair attention, little was done in class lectures to indicate the wide range of applicability of the subject to various situations in life.

MISS V. THAKURDAS (Lahore) remarked that in the Punjab, applied mathematics was overshadowed by pure mathematics.

W. A. DHANI (Student, Aligarh) said that education must not be divorced from the bread and butter problem.

Winding up the debate, SIR ZIA-UD-DIN AHMAD surveyed the various points discussed and said that teaching should not be subordinated to examinations. He thought that a group of mathematicians might be asked to set standard question papers.

Thanking Dr. Zia-ud-din, DR. VAIDYANATHASWAMI (Madras) said there had been a full discussion of important aspects of teaching. It was necessary to decide the place and degree to which we should insist on a knowledge of subjects like statistics, vector analysis in a general university course.

MEMBERS PRESENT AT THE CONFERENCE AT ALIGARH

S. Abbas Hassan Razvi, M.A., Hyderabad (Deccan).
M. Abdulla Butt, M.A., Aligarh.
S. K. Abhyankar, M.SC., L.T., Gwalior.
Afzal Ahmad, M.A., Hyderabad (Deccan).
Akhury Vindyachal Prasad, M.Sc, Chapra (Bihar).
F. C. Auluck, M.A., Lahore.
Banerji, Ekanath, M.A., Cawnpore.
Banwari Lal, M.A., Meerut.
P. B. Bhattachari, M.SC., Delhi.
K. R. Gunjikar, B.SC., M.A., Bombay.
H. L. Gupta, M.SC., Cawnpore.
Hansraj Gupta, M.A., Ph.D., Hoshiapur.
S. M. Kerawala, M.A. (Cantab.), Aligarh.
M. K. Keval Ramani, M.A., Karachi.
R. *Krishnamoorti*, M.A., *Hyderabad*.
A. M. Kureishy, M.A., Aligarh.
F. W. Levi, Dr. Phil. Nat., Calcutta.
D. J. Madan, M.SC., LL.B., Karachi.
C. P. S. Menon, M.A., M.SC. (London), Dehra-Dun.
P. C. Mittal, M.A., B.A. (Hons.), Lucknow.
D. N. Moghe, M. SC., Bombay.
Mohammad Shabbar, M.SC., Aligarh.
A. Narasinga Rao, M.A., D.SC., L.T., Annamalainagar.
M. N. Parischa, M.SC., Lucknow.
S. S. Pillai, M.SC., Trivandram.
Qazi K. Ahmad, M.SC., Allahabad.
Ram Ballabh, M.SC., Lucknow.
Ram Behari, M.A., Ph.D., Delhi.
Ram Dhar Misra, Ph.D., Lucknow.
V. Rangachariar, M.SC., Patna.
M. Raziudin Siddiqi, Aligarh.
Sahib Ram, M.A., Lahore.
P. N. Saxena, M.A., Lucknow.
B. R. Seth, D.SC., Delhi.
S. M. Shah, M.A., Aligarh.
P. N. Sharma, *M.A., Delhi*.
S. W. Shiveshawarkar, M.A. (Cantab.), B.SC., I.C.S., Delhi.
P. D. Shukla, M.A., Lucknow.
A. M. Singh, Lucknow.
C. N. Sreenivasiengar, D.SC., Bangalore.
R. S. Varma, D.SC., Lucknow.
R. Vaidyanathaswamy, M.A., Ph.D., D.SC., Madras.
Zia-ud-din Ahmad, M.A. (Cantab.), Ph.D., D.SC., Aligarh.